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THE PROPERTIES OF GENERALIZED SOLUTIONS OF CAUCHY PROBLEM FOR THE HEAT EQUATIONS WITH A RANDOM RIGHT PART

The subject of this work is at the intersection of two branches of mathematics: mathematical physics and stochastic processes. The physical formulation of problems of mathematical physics with random factors was studied by Kampe de Feriet. In the works by E. Beisenbaev, Yu.V. Kozachenko and V.V. Buldygin a new approach studying the solutions of partial differential equations with random initial conditions was proposed. The authors investigate the convergence in probability of the sequence of function spaces of partial sums approximating the solution of a problem. The mentioned approach was used in the works by E. Barrasa de La Krus, Endzhyrgly, Ya.A. Kovalchuk. In the paper by V.V. Buldygin and Yu.V. Kozachenko the application of the Fourier method for the homogeneous hyperbolic equation with Gaussian initial conditions is justified and existence conditions in terms of correlation functions are studied. Homogeneous hyperbolic equation with random initial conditions from the space $Sub_{\varphi}(\Omega)$ are considered in works by B. V. Dovgay, G.I. Slyvka-Tylyshchak. The model of a solution of a hyperbolic type equation with random initial conditions was investigated in the papers by G.I. Slyvka-Tylyshchak. There is studied a boundary value problem of mathematical physics for the inhomogeneous hyperbolic equation with φ-subgaussian in right part in works by B. V. Dovgay. The parabolic type equations of Mathematical Physics with random factors of Orlicz spaces have been studied in the papers by Yu.V. Kozachenko and K.J. Veresh. Properties of the classical solution of the heat equation on a line with a random right part are considered in works by Yu.V. Kozachenko and G.I. Slyvka-Tylyshchak.

We consider a Cauchy problem for the heat equations with a random right part. We study the inhomogeneous heat equation on a line with a random right part. We consider the right part as a random function of the space $L_p(\Omega)$. The conditions of existence with probability one generalized solution of the problem are investigated.

Keywords: heat equation, stochastic processes, generalized solution.

1. Introduction. The influence of random factors should often be taken into account in solving problems of mathematical physics. The heat equation with random factors is a classical problem of the parabolic type of mathematical physics. In this paper the heat equation with random right part are examined. In particular, we give conditions of existence with probability one generalized solutions in the case when the right part is a random field, sample continuous with probability one from the space $L_p(\Omega)$.

The paper consists of the introduction and one parts. In the section we consider heat equations with random right part. For such problem conditions of existence with probability one of generalized solution with random right-hand side from the space $L_p(\Omega)$ are found.

2. Main Results. We consider the Cauchy problem for the heat equation [3]

$$\frac{\partial u(x,t)}{\partial t} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2} + \xi(x,t), \tag{1}$$

$$-\infty < x < +\infty, \quad t > 0,$$

subject to the initial condition

$$u(x,0) = 0, \quad -\infty < x < +\infty. \tag{2}$$

Let the function $\xi(x,t) = \{\xi(x,t), \ \ x \in R, \ t > 0\}$ is a random field, selective and continuous with probability one from the space $L_p(\Omega)$, such that $E\xi(x,t) = 0$, $E(\xi(x,t))^2 < +\infty$. Let us denote

$$B(x,t,z,s) = E\xi(x,t)\xi(y,s).$$

Let $B(x,t,z,s)$ be a continuous function.

Problem when the function $\xi(x,t)$ nonrandom has been seen in [3]. Consider

$$G(y,t) = \frac{1}{\sqrt{2\pi}} \int\limits_0^t e^{-a^2 y^2 (t-\tau)} \tilde{\xi}(y,\tau) d\tau,$$

$$\tilde{\xi}(y,\tau) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \cos yx \xi(x,\tau) dx,$$

and

$$u(x,t) = \int\limits_{-\infty}^{+\infty} \cos yx G(y,t) dy. \tag{3}$$

Let $D = \{(x,t): \ x \in [-A, \ A], \ t \in [0, \ T]$ and

$$u_n(x,t) = \int\limits_{-a_n}^{+a_n} \cos yx G(y,t) dy, \tag{4}$$

where $a_n \to \infty$ for $n \to \infty$.

Definition 1. *The solution $u(x,t)$ which is represented in the form (3) is called a generalized solution of problem (1)—(2) in the domain D if sequence (4) converges uniformly in probability and satisfies the condition (2).*

Lemma 1. *[7] Let $\xi(x,t)$ is a random field, sample continuity for each $t > 0$ with probability one, there is a continuous derivative $\frac{\partial \xi(x,t)}{\partial x}$ for $x \in R$ and satisfy condition*

$$\int\limits_R \sqrt{E(\xi^2(x,t))} dx < \infty. \tag{5}$$

Then for the function $\xi(x,t)$ for each $t > 0$ the integral Fourier transform

$$\tilde{\xi}(y,\tau) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \cos yx \xi(x,\tau) dx$$

exist and

$$\xi(x,t) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \cos yx \tilde{\xi}(y,\ t) dy.$$

Lemma 2. *[8] Let $\xi(x,t)$ be a random field, sample continuity from the space $L_p(\Omega)$. Let $B(x,t,v,s)$ be the correlation function of the field $\xi(x,t)$. For all $t > 0,\ s > 0$ assume that $\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} |B(x,t,v,s)|\, dxdx \le B < \infty$.*

Then Lebesgue integrals

$$\int\limits_{-\infty}^{+\infty} \cos yx G(y,t) dy$$

exist with probability one.

Theorem 1. *[5] Let R^k be the k-dimensional space,*

$$d\,(t,s) = \max_{1 \leqslant i \leqslant k} |t_i - s_i|\,,$$

$T = \{0 \leqslant t_i \le T_i,\ i = 1, 2, ..., k\}\,,\ T_i > 0$. Let $X_n = \{X_n\,(t)\,,\ t \in T\}$ be a sequence of stochastic processes belonging to the Orlicz space $L_U(\Omega)$, and let the function U satisfy the g-condition. Assume that

1) *for all $t \in T$*

$$X_n(t) \to X(t)$$

in probability;

2)

$$\sup_{d(t,s) \le h}\ \sup_{n=\overline{1,\infty}} \|X_n(t) - X_n(s)\| \le \sigma(h),$$

where $\sigma = \{\sigma(h), h > o\}$ is a monotone increasing continuous function $\sigma(h) \to 0$ as $h \to 0$;

3) *for some $\varepsilon > 0$*

$$\int_0^{\varepsilon} u^{(-1)} \left(\prod_{i=1}^{k} \left(\frac{T_i}{2\sigma^{(-1)}(u)} + 1 \right) \right) du < \infty,$$

where $\sigma^{(-1)}(u)$ is the inverse function to $\sigma(u)$.

Then the processes $X_n(t)$ are continuous with probability one and the converge in probability in the space $C(T)$.

Theorem 2. *[7] Let $\widetilde{T} = \{-A \le x \le A,\ 0 \le t \le T\}$*

$$d\,(x, x_1, t, t_1) = \max_{1 \leqslant i \leqslant k} \left(|x - x_1|\,, |t - t_1|\right).$$

$X_n = \{X_n\,(x,t)\,,\ (x,t) \in T\}$ be a sequence of stochastic processes belonging to the space $L_p(\Omega)$, where $p > 2$. The sequence $X_n(x,t)$ converges in probability in the space $C(\widetilde{T})$ if

1) for all $(x,t) \in T$ $X_n(x,t) \to X(x,t)$ in probability;

2)
$$\sup_{d(x,x_1,t,t_1)\le h}\ \sup_{n=\overline{1,\infty}} \left(E\left|X_n(x,t)-X_n(x_1,t_1)\right|^p\right)^{\frac{1}{p}} \le Ch^{\alpha},$$
where $\alpha > \frac{2}{p}$.

Lemma 3. *[5] Let a function $X(\lambda,u)$, $\lambda>0$ and $u>0$ be such that:*

1) $\sup\limits_{0\le u<\infty, 0\le\lambda<\infty} |X(\lambda,u)| \le B;$

2) $|X(\lambda,u)-X(\lambda,v)| \le C\lambda|u-v|$ *for all* $u>0$, $v>0$.

Let $\varphi(\lambda)$, $\lambda>0$ be a continuous increasing function such that $\varphi(\lambda)>0$ for all $\lambda>0$, and the function $\frac{\lambda}{\varphi(\lambda)}$ is increasing for $\lambda>v_0$, and for some constant $v_0\ge 0$. Then
$$|X(\lambda,u)-X(\lambda,v)| \le \max(C,2B)\frac{\varphi(\lambda+v_0)}{\varphi\left(\frac{1}{|u-v|}+v_0\right)}$$
for all $\lambda\ge 0$ and $v>0$.

Theorem 3. *Let $\xi(x,t)$ is a random field, selective and continuous with probability one from the space $L_p(\Omega)$. and the conditions of lemma 1 and lemma 2 are holds, and*
$$\int\limits_{-\infty}^{+\infty} \left(E\left|\xi(x,\tau)\right|^p\right)^{\frac{1}{p}} dx < \Theta; p>2.$$
Then $u(x,t)$ is the classical solution to the problem (1)–(2).

Proof. It follows from Lemma 2 that there exist with probability one integral
$$\int\limits_{-\infty}^{+\infty} \cos yxG(y,t)dy,$$
Then from Theorem 1 that integral (4) are converging in probability to integral
$$\int\limits_{-\infty}^{+\infty} \cos yxG(y,t)dy,$$
for $|x|\le A$, $0\le t\le T$.

By to Theorem 2, integral (4) coincide in probability space $C(\widetilde{T})$ required that the conditions:
$$\left(E\left|u_{a_n}(x,t)-u_{a_n}(x_1,t_1)\right|^p\right)^{\frac{1}{p}} \le Ch^{\alpha},$$

where
$$u_{a_n}(x,t) = \int\limits_{-a_n}^{a_n} \cos yxG(y,t)dy.$$

Using generalized Minkovskoho inequality we obtain

$$(E\left|u_{a_n}(x,t)-u_{a_n}(x_1,t_1)\right|^p)^{\frac{1}{p}} =$$

$$\left(E\left|\int\limits_{-a_n}^{a_n}\cos yxG(y,t)dy-\int\limits_{-a_n}^{a_n}\cos yx_1G(y,t_1)dy\right|^p\right)^{\frac{1}{p}} =$$

$$\left(E\left|\int\limits_{-a_n}^{a_n}\left[\cos yxG(y,t)-\cos yx_1G(y,t_1)\right]dy\right|^p\right)^{\frac{1}{p}} =$$

$$\left(E\left|\int\limits_{-a_n}^{a_n}\left[(\cos yx-\cos yx_1)G(y,t_1)+(G(y,t)-G(y,t_1))\cos yx\right]dy\right|^p\right)^{\frac{1}{p}} \leq$$

$$\int\limits_{-\infty}^{\infty}\left[\left|\cos yx-\cos yx_1\right|(|G(y,t_1)|^p)^{\frac{1}{p}}+(E|G(y,t)-G(y,t_1)|^p)^{\frac{1}{p}}\right]dy.$$

Using the inequality $|\sin x|\leq|x|^{\alpha}$ for $0<\alpha\leq 1$ and an arbitrary h, $|x-x_1|\leq h$ we will have

$$|\cos yx-\cos yx_1|\leq 2\left|\sin\frac{y(x-x_1)}{2}\right|\leq 2^{1-\alpha}|y|^{\alpha}h^{\alpha}.$$

Consider

$$(E|G(y,t_1)|^p)^{\frac{1}{p}}=\frac{1}{\sqrt{2\pi}}\left(E\left|\int\limits_0^{t_1}e^{-a^2y^2(t_1-\tau)}\tilde{\xi}(y,\tau)d\tau\right|^p\right)^{\frac{1}{p}}\leq$$

$$\frac{1}{\sqrt{2\pi}}\int\limits_0^{t_1}e^{-a^2y^2(t_1-\tau)}\left(E|\tilde{\xi}(y,\tau)|^p\right)^{\frac{1}{p}}d\tau.$$

$$\left(E|\tilde{\xi}(y,\tau)|^p\right)^{\frac{1}{p}}=\frac{1}{\sqrt{2\pi}}\left(E\left|\int\limits_{-\infty}^{+\infty}\cos yx\xi(x,\tau)dx\right|^p\right)^{\frac{1}{p}}\leq$$

$$\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\left(E\left|\xi(x,\tau)\right|^p\right)^{\frac{1}{p}}dx<\frac{1}{\sqrt{2\pi}}\Theta.$$

Therefore

$$(E|G(y,t_1)|^p)^{\frac{1}{p}}\leq\frac{1}{2\pi}\int\limits_0^{t_1}\Theta e^{-a^2y^2(t_1-\tau)}d\tau\leq$$

$$\frac{1}{2\pi}\Theta^{\frac{1}{p}}\frac{1}{a^2y^2}\left|1-e^{-a^2y^2t_1}\right|.$$

Let $t_1 < t$ then

$$(E|G(y,t)-G(y,t_1)|^p)^{\frac{1}{p}} =$$

$$\frac{1}{\sqrt{2\pi}}\left(E\left|\int_0^t e^{-a^2y^2(t-\tau)}\tilde{\xi}(y,\tau)d\tau - \int_0^{t_1} e^{-a^2y^2(t_1-\tau)}\tilde{\xi}(y,\tau)d\tau\right|^p\right)^{\frac{1}{p}} =$$

$$\frac{1}{\sqrt{2\pi}}\left(E\left|\int_0^{t_1}\left[e^{-a^2y^2(t-\tau)} - e^{-a^2y^2(t_1-\tau)}\right]\tilde{\xi}(y,\tau)d\tau + \int_{t_1}^t e^{-a^2y^2(t-\tau)}\tilde{\xi}(y,\tau)d\tau\right|^p\right)^{\frac{1}{p}} =$$

$$\frac{1}{\sqrt{2\pi}}\left(\int_0^{t_1}\left[\left|e^{-a^2y^2(t-\tau)} - e^{-a^2y^2(t_1-\tau)}\right|\left(E|\tilde{\xi}(y,\tau)|^p\right)^{\frac{1}{p}}\right]d\tau + \right.$$

$$\left.\int_{t_1}^t e^{-a^2y^2(t-\tau)}\left(E|\tilde{\xi}(y,\tau)|^p\right)^{\frac{1}{p}}d\tau\right).$$

Using Lemma 3, we obtain the estimate

$$\left|e^{-a^2y^2(t-\tau)} - e^{-a^2y^2(t_1-\tau)}\right| = \left|e^{-a^2y^2(t_1-\tau)}\right|\left|e^{-a^2y^2(t-t_1)} - 1\right| \le$$

$$e^{-a^2y^2(t_1-\tau)}\max(1,a^2)y^{2\alpha}|t-t_1|^\alpha \le e^{-a^2y^2(t_1-\tau)}\max(1,a^2)y^{2\alpha}h^\alpha.$$

Therefore

$$(E|G(y,t)-G(y,t_1)|^p)^{\frac{1}{p}} \le$$

$$\frac{1}{2\pi}\left(\int_0^t e^{-a^2y^2(t_1-\tau)}\max(1,a^2)y^{2\alpha}h^\alpha\Theta^{\frac{1}{p}}d\tau + \int_{t_1}^t e^{-a^2y^2(t-\tau)}\Theta^{\frac{1}{p}}d\tau\right) =$$

$$\frac{\Theta}{2\pi}\left(\max(1,a^2)y^{2\alpha}h^\alpha\frac{1}{a^2y^2}\left|1-e^{-a^2y^2t_1}\right| + \int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right) =$$

$$\frac{\Theta}{2\pi}\left(\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right| + \int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right).$$

Then

$$(E\left|u_{a_n}(x,t)-u_{a_n}(x_1,t_1)\right|^p)^{\frac{1}{p}} \le$$

$$\frac{\Theta}{2\pi}\int_{-\infty}^{+\infty}\left[2^{1-\alpha}|y^\alpha h^\alpha|\frac{1}{a^2y^2}\left|1-e^{-a^2y^2t_1}\right| + h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right| + \right.$$

$$\left.\int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right]dy = \frac{\Theta}{\pi}\int_0^{+\infty}\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right| + \right.$$

$$\left.h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right| + \int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right]dy =$$

$$\frac{\Theta}{\pi}\left\{\int_0^1\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right| + h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha}\left|1-e^{-a^2y^2t_1}\right)\right|\right.$$

$$\left.+\int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right]dy+\int_1^{+\infty}\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|+\right.$$

$$\left.\left.h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right|+\int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right]dy\right\}=\frac{\Theta}{\pi}\left(I_1+I_2\right).$$

Consider

$$I_1=\int_0^1\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right| + h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right|\right.$$

$$\left.+\int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right]dy=\frac{2^{1-\alpha}h^\alpha}{a^2}\int_0^1\frac{1}{y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|dy+$$

$$\frac{h^\alpha}{a^2}\max(1,a^2)\int_0^1\frac{1}{y^{2(1-\alpha)}}\left|1-e^{-a^2y^2t_1}\right|dy+\int_0^1\left(\int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right)dy=$$

$$\frac{2^{1-\alpha}h^\alpha}{a^2}I_{11}+\frac{h^\alpha}{a^2}\max(1,a^2)I_{12}+I_{13}.$$

Since $\left|1-e^{-a^2y^2t_1}\le a^2y^2t_1\right|\le a^2y^2T$,

$$I_{11}=\int_0^1\frac{1}{y^{2(1-\alpha)}}\left|1-e^{-a^2y^2t_1}\right|dy\le\frac{a^2T}{\alpha+1}.$$

$$I_{12}=\int_0^1\frac{1}{y^{2(1-\alpha)}}\left|1-e^{-a^2y^2t_1}\right|dy\le\frac{a^2T}{2\alpha+1}.$$

Using that $e^{-a^2y^2(t-\tau)}\le 1$, we have

$$I_{13}=\int_0^1\left(\int_{t_1}^t e^{-a^2y^2(t-\tau)}d\tau\right)dy\le\int_0^1(t-t_1)dy\le h\le h^\alpha T^{1-\alpha}.$$

So we have

$$I_1\le h^\alpha\left(\frac{2^{1-\alpha}T}{\alpha+1}+\frac{\max(1,a^2)T}{2\alpha+1}+T^{1-\alpha}\right).$$

$$I_2=\int_1^{+\infty}\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|dy+\right.$$

$$h^\alpha \max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right| + \int\limits_{t_1}^{t} e^{-a^2y^2(t-\tau)}d\tau \Bigg] dy =$$

$$\frac{2^{1-\alpha}h^\alpha}{a^2}\int\limits_1^{+\infty}\frac{1}{y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|dy + \frac{h^\alpha}{a^2}\max(1,a^2)\int\limits_1^{+\infty}\frac{1}{y^{2(1-\alpha)}}\left|1-e^{-a^2y^2t_1}\right|dy +$$

$$\int\limits_1^{+\infty}\left(\int\limits_{t_1}^{t} e^{-a^2y^2(t-\tau)}d\tau\right)dy = \frac{2^{1-\alpha}h^\alpha}{a^2}I_{21} + \frac{h^\alpha}{a^2}\max(1,a^2)I_{22} + I_{23}.$$

$$I_{21} = \int\limits_1^{+\infty}\frac{1}{y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|dy \le \int\limits_1^{+\infty}\frac{1}{y^{2-\alpha}}dy = \frac{1}{1-\alpha}.$$

$$I_{22} = \int\limits_1^{+\infty}\frac{1}{y^{2(1-\alpha)}}\left|1-e^{-a^2y^2t_1}\right|dy \le \int\limits_1^{+\infty}\frac{1}{y^{2(1-\alpha)}}dy = \frac{1}{1-2\alpha}.$$

$$I_{23} = \int\limits_1^{+\infty}\left(\int\limits_{t_1}^{t} e^{-a^2y^2(t-\tau)}d\tau\right)dy = \frac{1}{a^2}\int\limits_1^{+\infty}\frac{1}{y^2}\left(1-e^{-a^2y^2(t-t_1)}\right)dy \le$$

$$\frac{h^\alpha}{a^2}\max 1, a^2\int\limits_1^{+\infty}\frac{dy}{y^{2(1-\alpha)}} = \frac{h^\alpha}{a^2}\max(1,a^2)\frac{1}{1-2\alpha}.$$

Therefore

$$I_2 = \left(\frac{2^{1-\alpha}}{a^2}\cdot\frac{1}{1-\alpha} + \frac{2\max(1,a^2)}{a^2}\right)h^\alpha.$$

Then for $0 < \alpha < \frac{1}{2}$, we have

$$\left(E\left|u_{a_n}(x,t) - u_{a_n}(x_1,t_1)\right|^p\right)^{\frac{1}{p}} \le Ch^\alpha,$$

where

$$C = \frac{\Theta}{\pi}\left(\frac{2^{1-\alpha}T}{\alpha+1} + \frac{\max(1,a^2)T}{2\alpha+1} + T^{1-\alpha} + \frac{2^{1-\alpha}}{a^2}\cdot\frac{1}{1-\alpha} + \frac{2\max(1,a^2)}{a^2}\right).$$

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Сливка-Тилищак Г. І. Властивості узагальненого розв'язку задачі Коші для рівняння теплопровідності з випадковою правою частиною.

Тематика даної роботи знаходиться на перетині двох галузей математики: математична фізика та випадкові процеси. Умови та оцінки швидкості збіжності за ймовірністю випадкових рядів знайшли широке застосування при розв'язувані задач математичної фізики з випадковими факторами. Фізичні постановки таких задач розглядав Кампе де Фер'є. У роботах Ю. В. Козаченка, В. В. Булдигіна, Є. Бесенбаєва запропоновано підхід, який ґрунтується на дослідженні у певних функціональних просторах, збіжності за ймовірністю послідовності часткових сум, що апроксимують розв'язки деяких крайових задач. В. В. Булдигін та Ю. В. Козаченко розглядали першу крайову задачу для однорідного гіперболічного рівняння з випадковими гауссовими початковими умовами. В роботах Ю. В. Козаченка і Барраса де Ла Крус ця ж задача вивчалась, коли початкові умови є випадковими процесами з просторів Орліча. У багатовимірному випадку однорідне гіперболічне рівняння з початковими випадковими умовами з простору $Sub_{\varphi}(\Omega)$ розглядалась в роботах Ю. В. Козаченка і Г. І. Сливки. У працях Б. В. Довгая вперше розглядалась крайова задача для неоднорідних рівнянь з випадковою правою частиною, що є φ-субгауссовим випадковим полем. Параболічні рівняння математичної фізики з випадковими факторами з простору Орліча також досліджувалися у роботах Ю. В. Козаченка та К. Й. Вереш. Властивості класичного розв'язку рівняння теплопровідності на прямій досліджувався у працях Ю.В. Козаченка та Г.І. Сливка-Тилищак.

В роботі розглянуто задачу Коші для рівняння теплопровідності на прямій з випадковою правою частиною з простору $L_p(\Omega)$. Отримано умови існування з імовірністю одиниця узагальненого розв'язку такої задачі.

Ключові слова: рівняння теплопровідності, випадкові процеси, узагальнений розв'язок.

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