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ON QUASI-PRIME DIFFERENTIAL SEMIRING IDEALS

The notion of a `quasi-prime ideal`, for the first time, was introduced in differential commutative rings, i.e. commutative rings considered together with a derivation, as differential ideals maximal among those not meeting some multiplicatively closed subset of a ring. The notion of a semiring derivation is traditionally defined as an additive map satisfying the Leibnitz rule. Due to rapid development of semiring theory in recent years, the need of considering ideals in semirings defined by similar conditions arose.

The present paper is devoted to investigating the notion of a `quasi-prime ideal` of differential semiring (which is defined as a semiring together with a derivation on it), not necessarily commutative. It aims to show, how `quasi-prime ideals` are related to prime differential ideals, primary ideals, maximal ideals and other types of ideals of semirings. The paper consists of two main parts. In the first part, the author investigates some properties of quasi-prime differential ideals, and gives some examples of such semiring ideals, such as prime differential, maximal differential ideals, or ideal obtained by derivation operator acting on a prime ideal of a semiring. It contains a theorem, which gives equivalent conditions for a quasi-prime semiring ideal to be prime.

The second part of the paper is devoted to considering chains of `quasi-prime ideals`. In this part, the interrelation between `quasi-prime ideals` and other types of differential ideals of semirings is established. It contains a theorem, which gives a characterization of such ideals in case of a commutative semiring. This characterization involves the notion of the radical of an ideal of a semiring and a derivation operator for semirings. The paper ends with a theorem, which states that every chain of quasi-prime ideals of a semiring has the least upper bound and the greatest lower bound. It is also proven that every `quasi-prime ideal` containing some differential ideal contains a `quasi-prime ideal` minimal among all the quasi-prime ideals of the given semiring, which contain the above mentioned differential ideal.

Keywords: differential semiring, differential ideal, semiring ideal, quasi-prime ideal.

1. Introduction. Semirings were introduced by Vandiver [9] as a generalization of associative rings and distributive lattices. The notion of a semiring derivation is defined in [4] as an additive map satisfying the Leibnitz rule. Thierrin [8] studied a semiring of languages over some alphabet and showed that it forms a differential additively idempotent semiring under the operations of union as the addition and catenation as the product, proving that differential semirings are of great interest due to their possible applications. Recently Chandramouleeswaran and Thiruveni [2] investigated different properties of semiring derivations and differential semiring ideals. This motivates a further study into properties of differential semirings, not necessarily idempotent, commutative, or connected with formal languages. Quasi-prime ideals were introduced by Keigher [6] for differential commutative rings. The objective of this paper is to investigate quasi-prime ideals of differential semirings, not nesessarily commutative, and their interrelation with prime differential ideals.

For the sake of completeness some definitions and properties used in the paper will be given here. For more information on semirings see [4] or [5].

Let R be a nonempty set, and let $+$ and $\cdot$ be binary operations on R. An algebraic system $(R, +, \cdot)$ is called a *semiring* if $(R, +, 0)$ is a commutative monoid, $(R, \cdot)$ is a semigroup and multiplication distributes over addition from either side. A semiring $(R, +, \cdot)$ is said to be *commutative* if $\cdot$ is commutative on R.

Zero $0 \in R$ is called (*multiplicatively*) *absorbing* if $a \cdot 0 = 0 \cdot a = 0$ for all $a \in R$. An element $1 \in R$ is called an *identity* if $a \cdot 1 = 1 \cdot a = a$ for all $a \in R$.

An element $a \in R$ is called *additively cancellable* if $a + b = a + c$ follows $b = c$ for all $b, c \in R$. Denote by $K^+(R)$ the set of all additively cancellable elements of R. A semiring R is called *additively cancellative* if $K^+(R) = R$.

An element $a \in R$ is called *additively idempotent* if $r + r = r$. Denote by $I^+(R)$ the set of all additively idempotent elements of R. A semiring R is called *additively idempotent* if $I^+(R) = R$.

A semiring is called *entire* if $ab = 0$ implies that either $a = 0$ or $b = 0$ for all $a, b \in R$. A subset S of R closed under addition and multiplication is called a *subsemiring* of R. The *center* of a semiring R is a set $Z(R) = \{r \in R | rs = sr \text{ for all } s \in R\}$. It is a subsemiring of R. Since $0 \in Z(R)$, $Z(R) \neq \varnothing$.

A *left ideal* of a semiring R is a nonempty set $I \neq R$ which is closed under addition and satisfies the condition $ra \in I$ for all $a \in I$, $r \in R$. Similarly we can define a right ideal and a (two-sided ideal) of a semiring. An ideal I of a semiring R is called *subtractive* (or *k-ideal*) if $a \in I$ and $a + b \in I$ follow $b \in I$ for any $a, b \in R$. An ideal I of the semiring R is called *strong* if $a + b \in I$ implies $a \in I$ and $b \in I$ for any $a, b \in R$. Every strong ideal is subtractive. The *k-closure* $cl(I)$ of an ideal I is the set $cl(I)$ of all elements $a \in R$ such that $a + b \in I$ for some $b \in I$. It is an ideal of R satisfying $I \subseteq cl(I)$ and $cl(cl(I)) = cl(I)$. An ideal I of R is subtractive if and only if $I = cl(I)$.

The *zeroid* $Zr(R)$ of a semiring R is the set of elements a of R such that there exists $b \in R$ such that $a + b = b$. The zeroid of a ring consists of 0 only. The zeroid of a semiring is a (two-sided) ideal. [1]

A *prime ideal* of R is an ideal $P \neq R$ such that whenever $IJ \subseteq P$ for any ideals I and J of R then either $I \subseteq P$ or $J \subseteq P$. An ideal P of a commutative semiring R is prime if and only either $a \in P$ or $b \in P$ whenever $ab \in P$ for any $a, b \in R$. A *primary ideal* of a commutative semiring R is a proper ideal P of R for which either $a \in P$ or $b \in \sqrt{P}$ whenever $ab \in P$. An ideal P is primary if and only if $IJ \subseteq P$ implies that either $I \subseteq P$ or $J \subseteq \sqrt{P}$. If Q is a primary ideal of a commutative semiring R, then $\sqrt{Q}$ is a prime ideal of R [4].

Throughout the paper R denotes a semiring in the above sense, not nesessarily commutative, with identity 1 and absorbing zero $0 \neq 1$, unless stated otherwise. $\mathbb{N}$ denotes the set of positive integers, and $\mathbb{N}_0 = \mathbb{N} \bigcup \{0\}$.

2. Quasi-prime and prime differential ideals. A map $\delta \colon R \to R$ is called a *derivation* [4] on R if $\delta(a + b) = \delta(a) + \delta(b)$ and $\delta(ab) = \delta(a) b + a \delta(b)$ for any $a, b \in R$. A semiring R equipped with a derivation δ is called *differential* with respect to the derivation δ, or a *δ-semiring*, and denoted by (R, δ) [2].

For an element $a \in R$ denote $a^{(0)} = a$, $a' = \delta(a)$, $a'' = \delta(\delta(a))$, $\ldots a^{(n)} = \delta(a^{(n-1)})$, $n \in \mathbb{N}_0$, and $a^{(\infty)} = \{a^{(n)} | n \in \mathbb{N}_0\}$.

Let (R, δ) be a differential semiring. For a subset A of R we define its *differential*

$A_\#$ to be the set

$$A_\# = \left\{a \in R \,\middle|\, a^{(n)} \in A \,\text{for all}\, n \in \mathbb{N}_0\right\}.$$

An ideal I of the δ-semiring R is called *differential* [4] if $\delta(a) \in I$ whenever $a \in I$. A subsemiring S of the δ-semiring R is called *differential* if $a \in S$ follows $\delta(a) \in S$.

$\{0\}$ is a differential k-ideal of any differential semiring R. As noted in [2], in a differential semiring R with absorbing zero the set $V(R)$ of all additively invertible elements of R is a differential ideal.

The set $I^+(R)$ of all additively idempotent elements of R is a differential ideal of R. Every multiplicatively idempotent two-sided ideal I of a differential semiring R is differential. If I is a differential ideal of R, then its k-closure $cl(I)$ is a differential k-ideal of R [7].

Proposition 1. *The zeroid $Zr(R)$ of a differential δ-semiring R is a differential ideal of R.*

Proof. For $a \in Zr(R)$ solvability of the equation $a + x = x$ for $x \in R$ implies $\delta(a) + \delta(x) = \delta(x)$, which in turn gives $\delta(a) \in Zr(R)$.

Proposition 2. *If R is an additively cancellative differential semiring, then its center $Z(R)$ is a differential subsemiring of R.*

Proof. For $a \in Z(R)$ and $b \in R$ we have $ab = ba$. Then $\delta(ab) = \delta(a)b + \delta(b)a$ and $\delta(ba) = \delta(b)a + a\delta(b)$ follows $\delta(a)b = a\delta(b)$. Therefore, $\delta(a) \in Z(R)$.

A non-empty subset S of the semiring R is called an *m-system* [4] of R if for every $a, b \in S$ there exists an element $r \in R$ such that $arb \in S$. An ideal I of R is prime if and only if $R \setminus I$ is an m-system [4]. Any maximal ideal of a semiring is prime [4].

A differential ideal Q of R is called *quasi-prime* if it is maximal among differential ideals of R disjoint from some m-system of R.

Proposition 3. *Any prime differential ideal of R is quasi-prime.*

Proof. For any prime ideal P of R the complement $R \setminus P = S$ is an m-system [4]. The result follows by definition.

Proposition 4. *Every maximal differential ideal of R is quasi-prime.*

Proof. Let Q be a maximal among differential ideals of R, $S = U(R)$ be the set of units of R. Then S is an m-system and no differential ideal I contains a unit of R, so $Q \cap U(R) = \emptyset$. Therefore, Q is a quasi-prime ideal.

Proposition 5. *In any differential semiring R for any prime ideal P of R the differential ideal $P_\#$ is quasi-prime.*

Proof. Suppose P is a prime ideal of R and $S = R \setminus P$. Then S is an m-system and $S \cap P = \emptyset$. By Propositions 10 and 11 from [7], $P_\#$ is a differential ideal of R disjoint from S. If I is any differential ideal disjoint from S, then $I \subseteq P$. Thus $I = I_\# \subseteq P_\#$. Hence $P_\#$ is a quasi-prime ideal of R.

Theorem 1. *For a differential semiring R the following conditions are equivalent:*

1) Any quasi-prime ideal I in R is prime.

2) If I is a prime ideal of R, then $I_\#$ is a prime differential ideal of R.

3) Any prime ideal, minimal over some differential ideal, is differential.

4) If $S \subseteq R$ is an m-system of R ($0 \notin S$) and I is a differential ideal of R disjoint from S, then every differential ideal of R which is maximal among differential ideals containing I and not meeting S is prime.

Proof. (1) $\Longrightarrow$ (2) If I is prime then by Proposition 5 $I_{\#}$ is quasi-prime. Therefore, $I_{\#}$ is a prime differential ideal.

(2) $\Longrightarrow$ (3) Let I be a differential ideal of R, and let P be a prime ideal minimal among prime ideals containing I. Then $I = I_{\#} \subseteq P_{\#} \subseteq P$. Since $P_{\#}$ is prime, then $P_{\#} = P$. Therefore, P is a prime differential ideal.

(3) $\Longrightarrow$ (4) Obvious.

(2) $\Longrightarrow$ (4) Obvious.

(4) $\Longrightarrow$ (2) Suppose $S \subseteq R$ is an m-system of R ($0 \notin S$), I is a differential ideal of R such that $I \cap S = \varnothing$, and every differential ideal K of R, maximal among those containing I and not meeting S is prime. Let P be any prime ideal. Under given conditions $S = R \setminus P$ is an m-system of R and $\{0\}$ is a differential ideal disjoint from S. Moreover, $P_{\#} \subseteq P$ follows $S \cap P_{\#} = \varnothing$. Thus $P_{\#}$ is a differential ideal of R disjoint from S. If I is an arbitrary differential ideal of R such that $P_{\#} \subseteq I$ and $I \cap S = \varnothing$, then $I \subseteq P$. It follows that $I = I_{\#} \subseteq P_{\#}$. Thus $P_{\#}$ is prime.

3. Chains of quasi-prime ideals. Many interesting results on quasi-prime ideals can be obtained in commutative case.

Let A be a subset of R. Denote the smallest differential ideal containing the set A by $[A]$, the smallest radical differential ideal containing A by $\{A\}$, the smallest differential subtractive ideal containing the set A by $|A|$, and the smallest radical differential subtractive ideal containing A by $\langle A \rangle$.

Lemma 1. *Let $a, b \in R$, $n \in \mathbb{N}_0$, $\delta\colon R \longrightarrow R$ be a semiring derivation. Then $a^{n+1}\delta^n(b) \in |ab|$.*

Proof. By induction on n. The lemma is obviously true for $n = 0$. Let $n \geq 1$. Assume the assertion is true for all $k < n$.

Consider $\delta(a^n \cdot b^{(n-1)}) = na^{n-1}a'b^{(n-1)} + a^n b^{(n)}$, and multiply it by a. Then $a \cdot \delta(a^n \cdot b^{(n-1)}) = na^n a'b^{(n-1)} + a^{n+1}b^{(n)}$. By induction hypothesis, since $|ab|$ is a differential ideal, then $a \cdot \delta(a^n \cdot b^{(n-1)}) \in |ab|$, moreover $na^n a'b^{(n-1)} + a^{n+1}$. Therefore by subtractivity of $|ab|$, $a^{n+1}b^{(n)} \in |ab|$, as needed.

Theorem 2. *Every quasi-prime ideal of R is primary.*

Proof. Let Q be a quasi-prime ideal of R, and let $a \notin Q$ and $b^n \notin Q$ for all $n \in \mathbb{N}_0$. Prove that Q is primary by showing that $ab \notin Q$. There exists a multiplicatively closed subset S of R such that $S \cap Q = \emptyset$ and $I \cap Q \neq \emptyset$ for every $I \neq Q$ such that $Q \subset I$. Then $Q \subset Q + \sum_{k=1}^{n} R\delta^k(a) = I$ for some $n \in \mathbb{N}_0$ and $Q \neq I$, and by maximality of Q, $I \cap S \neq \emptyset$. So there exists $s \in S \cap Q + \sum_{k=1}^{n} R\delta^k(a)$ for some $m \in \mathbb{N}_0$. Similarly, there exists an element $t \in S \cap Q + \sum_{l=1}^{m} R\delta^l(b^{n+1})$. Then by Lemma 1 $s^r t \in |ab| + Q$ for some $r \in \mathbb{N}_0$. Hence $ab \notin Q$, for if $ab \in Q$, then $|ab| \subseteq Q$, and $|ab| + Q = Q$, so $s^r t \in Q$, and $S \cap Q \neq \emptyset$ which would contradict to the assumption.

In a commutative semiring R the *radical* of an ideal I is denoted by $\sqrt{I}$ and defined to be the set $\sqrt{I} = \{r \in R | r^n \in I \text{ for some } n \in \mathbb{N}_0\}$. According to [3] $I \subseteq \sqrt{I}$. If I is a subtractive ideal of R, then so is $\sqrt{I}$. Moreover, $\sqrt{I}$ is an intersection of all the prime ideals of R containing I, whenever $1 \in R$.

Theorem 3. *Let R be a commutative semiring. For a differential ideal Q of R the following conditions are equivalent:*
1) Q is quasi-prime;
2) Q is primary and $Q = (\sqrt{Q})_{\#}$;
3) $\sqrt{Q} \in Spec(R)$ and $Q = (\sqrt{Q})_{\#}$;
4) There exists $P \in Spec(R)$ such that $Q = P_{\#}$.

Proof. (1) $\Longrightarrow$ (2) Let Q be a quasi-prime ideal of R. Q is primary by Proposition 2. Moreover, Q is maximal among differential ideals of R disjoint from some multiplicatively closed subset S. Prove that $\sqrt{Q} \cap S = \emptyset$. If $a \in \sqrt{Q} \cap S$, then there exists $n \in \mathbb{N}_0$ such that $a^n \in Q$, and $a \in S$. Therefore, the $Ra^n \subseteq Q$, and $Ra^n \subseteq Q \cap S$, which contradicts to the assumption.

Since $Q \subseteq \sqrt{Q}$ then by [7] $Q = Q_{\#} \subseteq (\sqrt{Q})_{\#}$. Then $\sqrt{Q} \cap S = \emptyset$ and $(\sqrt{Q})_{\#} \subseteq \sqrt{Q}$ follow $(\sqrt{Q})_{\#} \cap S = \emptyset$. Since Q is the maximal among differential ideals of R not meeting S, then $Q = (\sqrt{Q})_{\#}$.

(2) $\Longrightarrow$ (3) Let Q be a primary ideal of R. Then $\sqrt{Q}$ is a prime ideal of R [4].

(3) $\Longrightarrow$ (4) $P = \sqrt{Q}$ is the prime ideal of R satisfying the condition $Q = P_{\#}$.

(4) $\Longrightarrow$ (1) Let P be a prime differential ideal of R such that $Q = P_{\#}$. Then by Proposition 5 Q is quasi-prime.

Proposition 6. *Let $f\colon R_1 \to R_2$ be a differential semiring homomorphism. If Q is a quasi-prime ideal of R_2, then $f^{-1}(Q)$ is a quasi-prime ideal of R_1.*

Proof. Let Q be a quasi-prime ideal of R_2. By Theorem 5 there exists $P \in Spec(R)$ such that $Q = P_{\#}$. Then $f^{-1}(Q) = f^{-1}(P_{\#}) = (f^{-1}(P))_{\#}$ by Proposition 13 [7]. Then again since $f^{-1}(P) \in Spec(R)$, by Theorem 3 $f^{-1}(Q)$ is quasi-prime in R_1.

Let $Spec(R)$ denote the spectrum of R. Denote by $Quas(R)$ the set of all quasi-prime differential ideals of R, and call it a quasi-prime spectrum of R. Then the map $\alpha\colon Spec(R) \to Quas(R)$ given by $\alpha(P) = P_{\#}$ for any $P \in Spec(R)$ is surjective, and the map $\beta\colon Quas(R) \to Spec(R)$ given by $\beta(Q) = \sqrt{Q}$ for any $Q \in Quas(R)$ is injective. Moreover, $\alpha\beta = id$ is the identity on $Quas(R)$.

A differential homomorphism $f\colon R_1 \to R_2$ induces a function $f^{-1}\colon Quas(R_2) \to Quas(R_1)$.

Theorem 4. *Let R be a commutative semiring. If $\{Q_i\}_{i\in I}$ is a chain of quasi-prime ideals of R, then $\bigcap_{i\in I} Q_i$ is a quasi-prime ideal of R and there is a unique smallest quasi-prime ideal of R containing $\bigcup_{i\in I} Q_i$.*

Every chain of quasi-prime ideals of R has the least upper bound and the greatest lower bound.

Proof. If $\{Q_i\}_{i\in I}$ is a chain of quasi-prime ideals of R, then by Proposition 2, $\{\sqrt{Q_i}\}_{i\in I}$ is a chain of prime ideals of R. Since $\bigcap_{i\in I} \sqrt{Q_i}$ and $\bigcup_{i\in I} \sqrt{Q_i}$ are prime ideals of R. By Proposition 10 from [7] $(\bigcap_{i\in I} \sqrt{Q_i})_{\#} = \bigcap_{i\in I} (\sqrt{Q_i})_{\#} = \bigcap_{i\in I} Q_i$. The ideal $\bigcap_{i\in I} \sqrt{Q_i}$ being prime follows that $(\bigcap_{i\in I} \sqrt{Q_i})_{\#}$ is quasi-prime, by Proposition 5, so is $\bigcap_{i\in I} Q_i$.

If Q is any quasi-prime ideal of R containing the prime ideal $\bigcup_{i\in I} Q_i$, then $\sqrt{\bigcup_{i\in I} Q_i} = \bigcup_{i\in I} \sqrt{Q_i} \subseteq \sqrt{Q}$. Thus $(\bigcup_{i\in I} \sqrt{Q_i})_{\#} \subseteq (\sqrt{Q})_{\#} = Q$.

Theorem 5. *Let R be a commutative semiring. Let I be a differential ideal of R and Q be a quasi-prime ideal of R such that $I \subseteq Q$. Then Q contains a quasi-prime* *ideal minimal among all quasi-prime ideals of R containing I.*

Proof. Embed Q in a maximal chain $\{Q_i\}_{i\in I}$ of quasi-prime ideals of R containing I. Thus, $\bigcap_{i\in I} Q_i$ is a quasi-prime ideal of R and is clearly minimal.

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Мельник І. О. Про квазіпервинні диференціальні ідеали напівкілець.

Поняття `квазіпервинного ідеалу` було вперше введено в комутативних диференціальних кільцях, тобто комутативних кільцях, які розглядаються разом із заданим на них диференціюванням, як диференціальний ідеал, максимальний серед диференціальних ідеалів, які не перетинаються із деякою мультиплікативно-замкненою підмножиною кільця. Поняття диференціювання у напівкільці традиційно визначають як адитивне відображення, яке задовольняє правило Лейбніца. У зв'язку з швидким розвитком теорії напівкілець в останні роки, виникла потреба у вивченні ідеалів, які визначаються подібними властивостями у напівкільцях.

Ця стаття присвячена дослідженню поняття `квазіпервинного ідеалу` в диференціальних напівкільцях (які означаються як напівкільця разом із диференціюванням, заданому на них), які не обов'язково комутативні. Метою статті є показати, як `квазіпервинні ідеали` пов'язані з первинними диференціальними ідеалами, примарними ідеалами, максимальними ідеалами та іншими типами ідеалів у напівкільцях. Стаття складається з двох основних частин. У першій частині автор досліджує деякі властивості квазіпервинних диференціальних ідеалів, а також подає деякі приклади таких ідеалів, зокрема первинні диференціальні, максимальні диференціальні та ідеали, які можна отримати в результаті дії оператора диференціювання на первинні ідеали напівкільця. У цій частині подано теорему, у якій даються еквівалентні умови того, що квазіпервинний ідеал є первинним.

У другій частині статті розглядаються ланцюги `квазіпервинних ідеалів`. У цій частині встановлено взаємозв'язки між `квазіпервинними ідеалами` та іншими типами диференціальних ідеалів напівкілець. В одній з теорем подано характеризацію таких ідеалів у випадку комутативних напівкілець. У цій характеризації використовуються поняття радикалу ідеалу напівкільця та оператор диференціювання в напівкільцях. На завершення статті подано теорему про те, що кожний ланцюг `квазіпервинних ідеалів` напівкільця має точну верхню і точну нижню межу. Також доведено, що кожний `квазіпервинний ідеал`, який містить деякий диференціальний ідеал, містить квазіпервинний ідеал, мінімальний серед усіх квазіпервинних ідеалів даного напівкільця, які містять вищезгаданий диференціальний ідеал.

Ключові слова: диференціальне напівкільце, диференціальний ідеал, ідеал напівкільця, квазіпервинний ідеал.

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