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ON TRANSITIVITY COEFFICIENTS FOR POSETS OF *MM*-TYPE TO BE OVERSUPERCRITICAL NON-PRIMITIVE

M. M. Kleiner proved that a poset S has finite representation type if and only if it does not contain subposets of the form

$$(1,1,1,1), \quad (2,2,2), \quad (1,3,3), \quad (1,2,5), \quad (\mathrm{N},4).$$

These posets are called the Kleiner's posets and are (up to isomorphism) all the critical posets relative to the finiteness of type (i.e. minimal posets having infinite representation type). Later Yu. A. Drozd proved that a poset S has finite representation type if and only if the quadratic form

$$q_S(z) =: z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j,i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i,$$

which is called the Tits quadratic form of S, is weakly positive (i.e., positive on the set of non-negative vectors). Thus, the Kleiner's posets are critical relative to weak positivity of the Tits quadratic form. In 2005 the authors proved that a poset is critical relative to the positivity of the Tits quadratic form if and only if it is minimax isomorphic to a Kleiner's poset.

An analogous situation takes place for posets of tame representation type. L. A. Nazarova proved that a poset S is tame if and only if it does not contain subsets of the form

$$(1,1,1,1,1), \quad (1,1,1,2), \quad (2,2,3), \quad (1,3,4), (1,2,6), \quad (\mathrm{N},5).$$

These posets are critical relative to weak non-negativity of the Tits quadratic form and are called supercritical.

In 2009 the authors proved that a poset is critical relative to non-negativity of the Tits quadratic form if and only if it is minimax isomorphic to a supercritical poset. The first author suggested to introduce so-called oversupercritical (or 1-oversupercritical) posets, which differ from the supercritical ones in the same degree as the supercritical posets differ from the critical ones. Among these posets, there is a single non-primitive poset, i.e. which is not a direct sum of chains. In this paper, we describe all posets that are minimax isomorphic to them and study some of their combinatorial properties. The importance of studying minimax isomorphic posets is determined by the fact that their Tits quadratic forms are $\mathbb{Z}$-equivalent, and minimax isomorphism itself is a fairly general constructively defined $\mathbb{Z}$-equivalence of the Tits quadratic forms for posets.

Keywords: representation, critical and supercritical poset, oversupercritical poset, Tits quadratic form, finite and tame representation type, positivity and non-negativity, transitivity coefficient.

1. Introduction. In [1] were introduced *1-oversupercritical posets* which differ from supercritical posets in the same degree as the latter differ from critical ones; often, including in this article, they are simply called *oversupercritical* (in more details see Introduction in [2]). Among these posets, there is, up to isomorphism, only one not being primitive (i.e. which is not a direct sum of chains), namely

$$U = \{1, 2, \ldots, 10 \,|\, 1 \prec 2 \prec 3 \prec 4 \prec 5 \prec 6, 7 \prec 8, 9 \prec 10, 7 \prec 10\}.$$

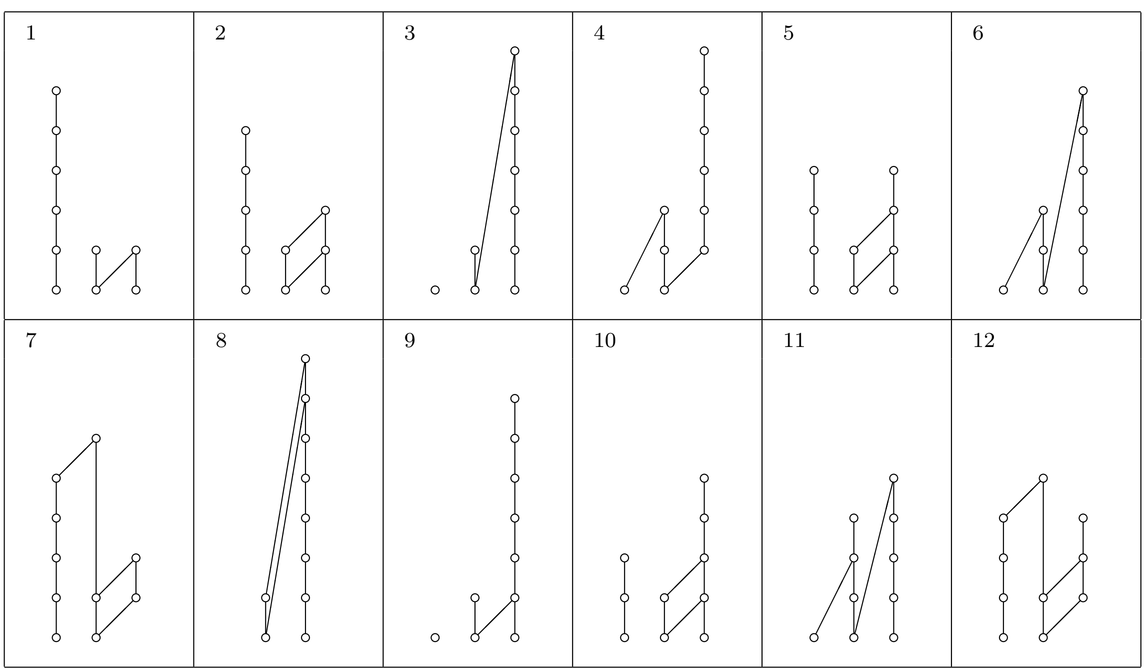
In this article, we describe all posets that are minimax isomorphic to him and study some of their combinatorial properties.

2. The main classification theorem. Let S be a finite poset. For an element $a \in S$ being minimal (resp. maximal), denote by $T = S_a^{\uparrow}$ (resp. $T = S_a^{\downarrow}$) the following poset: $T = S$ as usual sets, $T \setminus a = S \setminus a$ as posets, the element a is maximal (resp. minimal) in T, and a is comparable with x in T if and only if they are incomparable in S. Posets S and T are called (min, max)-*equivalent* or *minimax equivalent* if there are posets $S_1, \ldots, S_p$ ($p \geq 0$) such that, if we put $S = S_0$ and $T = S_{p+1}$, then, for every $i = 0, 1, \ldots, p$, either $S_{i+1} = (S_i)_{x_i}^{\uparrow}$ or $S_{i+1} = (S_i)_{y_i}^{\downarrow}$ [3]. Posets S and S' are called (min, max)-*isomorphic* or *minimax isomorphic* if there exists a poset X, which is minimax equivalent to S and isomorphic to S'. Obviously, empty poset is (min, max)-equivalent (and (min, max)-isomorphic) to itself.

Let P be a fix poset. A poset S is called of *MM-type* P if S is minimax isomorphic to P [4]. If P is oversupercritical we say that S is of *oversupercritical MM-type*. Posets of concrete MM-types were studied in many papers (see, besides the above mentioned papers, [5] – [13]). All posets of oversupercritical MM-type U are described by the following theorem (for a definition of U see Introduction).

Theorem 1. *Up to isomorphism and anti-isomorphism, the set of posets minimax isomorphic to U consists of the posets indicated in the following two tables.*

Table 1.



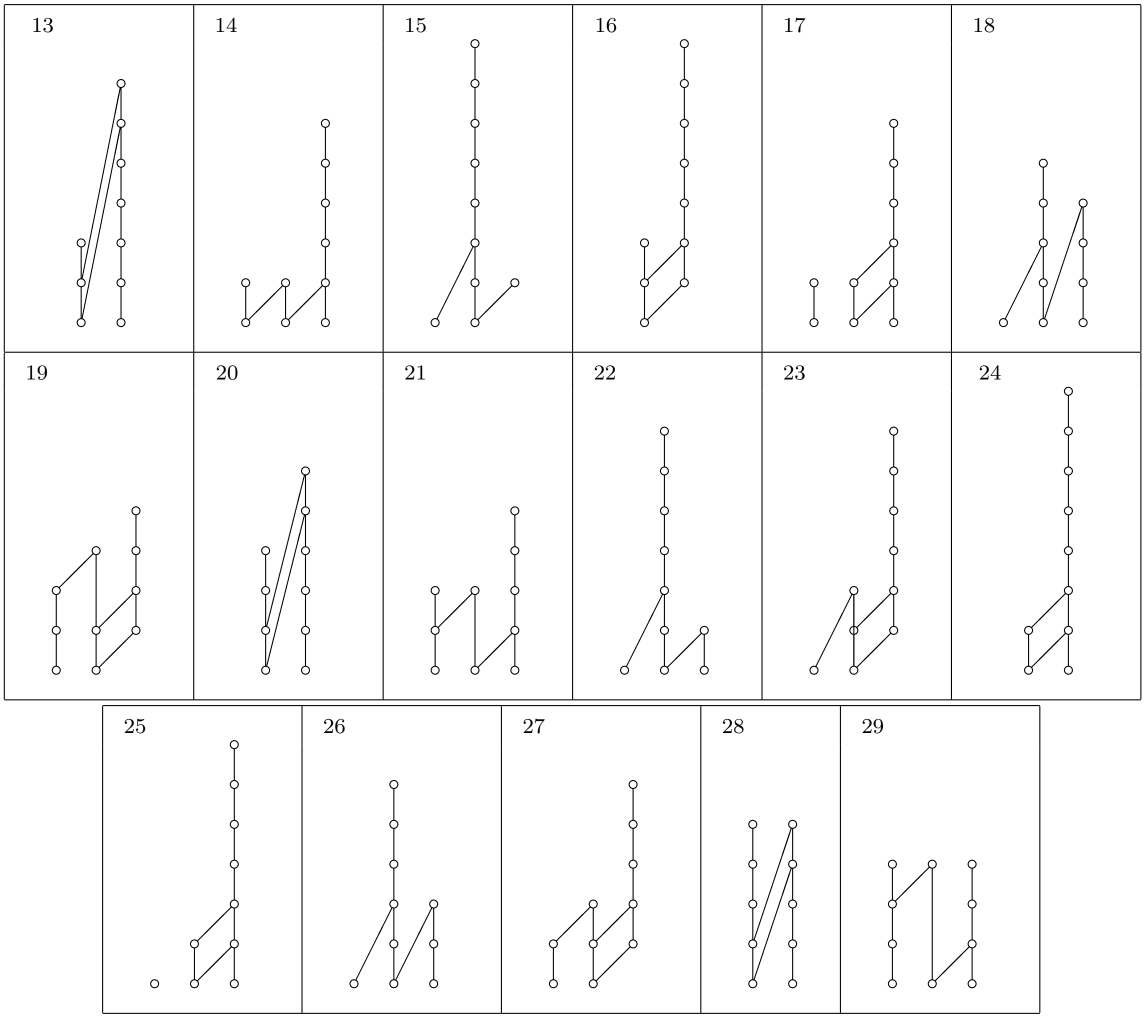
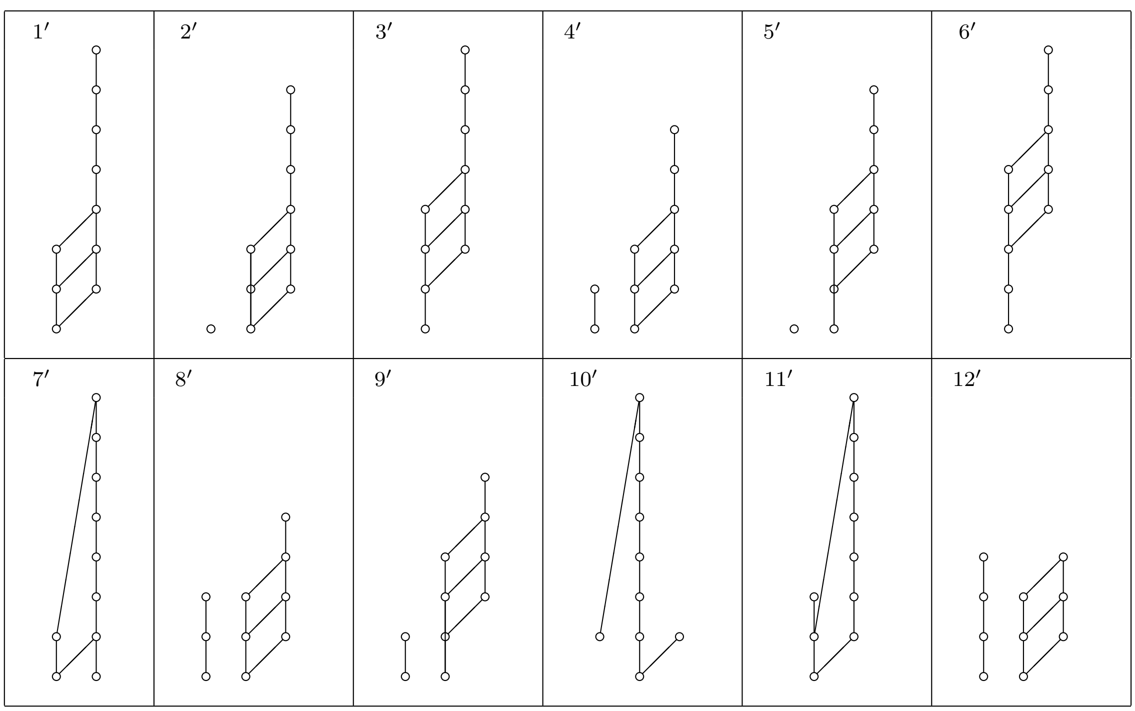


Table 2.



A poset with number s (resp.s'), indicated in Table 1 (resp. Table 2), is denoted by N_s (resp. $\mathrm{N}_{s'}$).

Recall that a poset T is called *dual to a poset* S and is denoted by S^{op} if $T = S$ as usual sets and $x < y$ in T if and only if $x > y$ in S. Posets S and T are called *anti-isomorphic* if S and T^{op} are isomorphic. Note that the posets N_x, $x = 1, 2, \ldots, 29, 1', 2' \ldots, 12'$ (indicated in Tables 1 and 2), are pairwise non-isomorphic and non-anti-isomorphic.

3. Proof of Theorem 1. We will use some ideas and definitions of [5], which are also presented in our paper published in the previous issue of this journal (see [2, Section 3]). Many of them (in particular, the algorithm for describing all posets that are minimax isomorphic to a given one) are not duplicated in this article.

Apply our algorithm to the proof of the theorem.

Step I. Describe (up to strongly isomorphic) all lower subposets X of the poset U. They are:
$X_1 = \varnothing$, $X_2 = \{1\}$, $X_3 = \{7\}$, $X_4 = \{9\}$, $X_5 = \{1,2\}$, $X_6 = \{1,7\}$, $X_7 = \{1,9\}$, $X_8 = \{7,8\}$, $X_9 = \{7,9\}$, $X_{10} = \{1,2,3\}$, $X_{11} = \{1,2,7\}$, $X_{12} = \{1,2,9\}$, $X_{13} = \{1,7,8\}$, $X_{14} = \{1,7,9\}$, $X_{15} = \{7,8,9\}$, $X_{16} = \{7,9,10\}$, $X_{17} = \{1,2,3,4\}$, $X_{18} = \{1,2,3,7\}$, $X_{19} = \{1,2,3,9\}$, $X_{20} = \{1,2,7,8\}$, $X_{21} = \{1,2,7,9\}$, $X_{22} = \{1,7,8,9\}$, $X_{23} = \{1,7,9,10\}$, $X_{24} = \{7,8,9,10\}$, $X_{25} = \{1,2,3,4,5\}$, $X_{26} = \{1,2,3,4,7\}$, $X_{27} = \{1,2,3,4,9\}$, $X_{28} = \{1,2,3,7,8\}$, $X_{29} = \{1,2,3,7,9\}$. $X_{30} = \{1,2,7,8,9\}$, $X_{31} = \{1,2,7,9,10\}$, $X_{32} = \{1,7,8,9,10\}$, $X_{33} = \{1,2,3,4,5,6\}$, $X_{34} = \{1,2,3,4,5,7\}$, $X_{35} = \{1,2,3,4,5,9\}$, $X_{36} = \{1,2,3,4,7,8\}$, $X_{37} = \{1,2,3,4,7,9\}$, $X_{38} = \{1,2,3,7,8,9\}$, $X_{39} = \{1,2,3,7,9,10\}$, $X_{40} = \{1,2,7,8,9,10\}$, $X_{41} = \{1,2,3,4,5,6,7\}$, $X_{42} = \{1,2,3,4,5,6,9\}$, $X_{43} = \{1,2,3,4,5,7,8\}$, $X_{44} = \{1,2,3,4,5,7,9\}$, $X_{45} = \{1,2,3,4,7,8,9\}$, $X_{46} = \{1,2,3,4,7,9,10\}$, $X_{47} = \{1,2,3,7,8,9,10\}$, $X_{48} = \{1,2,3,4,5,6,7,8\}$, $X_{49} = \{1,2,3,4,5,6,7,9\}$, $X_{50} = \{1,2,3,4,5,7,8,9\}$, $X_{51} = \{1,2,3,4,5,7,9,10\}$, $X_{52} = \{1,2,3,4,7,8,9,10\}$, $X_{53} = \{1,2,3,4,5,6,7,8,9\}$, $X_{54} = \{1,2,3,4,5,6,7,9,10\}$, $X_{55} = \{1,2,3,4,5,7,8,9,10\}$. Denote by K_i the poset $U^{\uparrow}_X$ for $X = X_i$ It was showed in [7] that, up to isomorphism and duality, the sets of posets K_i with i running from 1 to 55 coincides with the set of posets, indicated in Table 1.

Step II. Describe, up to strongly isomorphic, all pairs $Z = (Y, X)$ consisting of a proper lower subposet Y in U and a nonempty lower subposet X in Y such that $X < U \setminus Y$ (i.e. $x < t$ for any $x \in X, t \in U \setminus Y$). They are:
$Z_1 = (X_{32}, \{1\})$, $Z_2 = (X_{40}, \{1\})$, $Z_3 = (X_{40}, \{1,2\})$, $Z_4 = (X_{47}, \{1\})$, $Z_5 = (X_{47}, \{1,2\})$, $Z_6 = (X_{47}, \{1,2,3\})$, $Z_7 = (X_{49}, \{7\})$, $Z_8 = (X_{52}, \{1\})$, $Z_9 = (X_{52}, \{1,2\})$, $Z_{10} = (X_{52}, \{1,2,3\})$, $Z_{11} = (X_{52}, \{1,2,3,4\})$; $Z_{12} = (X_{53}, \{7\})$, $Z_{13} = (X_{53}, \{9\})$, $Z_{14} = (X_{53}, \{7,9\})$, $Z_{15} = (X_{54}, \{7\})$, $Z_{16} = (X_{55}, \{1\})$, $Z_{17} = (X_{55}, \{1,2\})$, $Z_{18} = (X_{55}, \{1,2,3\})$, $Z_{19} = (X_{55}, \{1,2,3,4\})$, $Z_{20} = (X_{55}, \{1,2,3,4,5\})$.

Denote by K'_i the poset $(U^{\uparrow}_Y)^{\uparrow}_X$ for $(Y, X) = Z_i$ and show that, up to isomorphism and duality, the sets of posets K'_i with i running from 1 to 20 coincides with the set of posets, indicated in Table 2. Indeed, it is easy to see that $K'_1 \cong N^{op}_{1'}$, $K'_2 \cong N^{op}_{2'}$, $K'_3 \cong N^{op}_{3'}$, $K'_4 \cong N^{op}_{4'}$, $K'_5 \cong N^{op}_{5'}$, $K'_6 \cong N_{6'}$, $K'_7 \cong N_{7'}$, $K'_8 \cong N^{op}_{8'}$, $K'_9 \cong N_{9'}$, $K'_{10} \cong N_{5'}$, $K'_{11} \cong N_{3'}$, $K'_{12} \cong N_{10'}$, $K'_{13} \cong N^{op}_{11'}$, $K'_{14} \cong N^{op}_{7'}$, $K'_{15} \cong N_{11'}$, $K'_{16} \cong N_{12'}$, $K'_{17} \cong N_{8'}$, $K'_{18} \cong N_{4'}$, $K'_{19} \cong N_{2'}$, $K'_{20} \cong N_{1'}$.

Step III. It is easy to verify that all the posets, indicated in the condition of the theorem, and dual to them (in the non-dual cases) occur in I and II (and even one time at a time). And hence the theorem is proved.

4. Coefficientts of transitivity. For a (finite) poset S, we put $S^2_< := \{(x,y)\,|\,x,y\in S, x<y\}$. If $(x,y)\in S^2_<$ and there is no z satisfying $x<z<y$, then we say that x and y are *neighboring*. We put $n_w = n_w(S) := |S^2_<|$ and denote by $n_e = n_e(S)$ the number of pairs of neighboring elements. The ratio $k_t = k_t(S)$ of the numbers $n_w - n_e$ and n_w is called by definition the *coefficient of transitivity* of S (see [10]). Note that in the case $n_w = 0$ (then $n_e = 0$) we assume $k_t = 0$.

In this part of the paper we calculate k_t for the posets of MM-type U.

Theorem 2. *The following holds for posets N_s, $s = 1,2,\dots,29,1',2'\dots,12'$:*

N	n_e	n_w	k_t	N	n_e	n_w	k_t	N	n_e	n_w	k_t
1	8	18	0,55556	11	9	19	0,52632	21	9	20	0,55
2	9	17	0,47059	12	10	21	0,52381	22	9	28	0,67857
3	8	23	0,65217	13	10	27	0,62963	23	10	30	0,66667
4	9	25	0,64	14	9	23	0,60870	24	10	42	0,76190
5	9	18	0,5	15	9	35	0,74286	25	9	33	0,72727
6	9	20	0,55	16	10	37	0,72973	26	9	23	0,60870
7	10	22	0,54545	17	9	26	0,65385	27	10	25	0,6
8	10	32	0,6875	18	9	20	0,55	28	10	23	0,56522
9	8	28	0,71429	19	10	22	0,54545	29	9	19	0,52632
10	9	21	0,57142	20	10	24	0,58333				

N	n_e	n_w	k_t	N	n_e	n_w	k_t	N	n_e	n_w	k_t
$1'$	11	42	0,73810	$5'$	10	33	0,69697	$9'$	10	26	0,61538
$2'$	10	33	0,69697	$6'$	11	42	0,73810	$10'$	9	30	0,7
$3'$	11	42	0,73810	$7'$	10	37	0,72973	$11'$	10	32	0,6875
$4'$	10	26	0,61538	$8'$	10	21	0,52381	$12'$	10	18	0,44444

The transitivity coefficients are written out with an accuracy of five decimal places. The value is exact if and only if the number of decimal places is less than five, and two values equal to exactly five digits are equal at all.

The proof is carried out by direct calculations.

Recall that *height of a poset S* is, by definition, the greatest length among the lengths of all linear ordered subsets of S. An element of a poset is called *nodal*, if it is comparable with all the other elements. A subposet X of T is called *dense* if there is not $x_1, x_2 \in X$, $y \in T\setminus X$ such that $x_1 < y < x_2$.

Note that a poset of MM-type U can have at most six nodal elements.

Corollary 1. *The coefficient $k_t(S)$ of a poset S is the largest among those for all the posets of MM-type U if and only if S contains a dense subposet with six nodal elements.*

Corollary 2. *The coefficient $k_t(S)$ of a poset S is the smallest among those for all the posets of MM-type U if and only if S is a self-dual non-connected poset of height four.*

Corollary 3. *For a posets S of MM-type U, the following conditions are equivalent:*

(a) $k_t(S) = \frac{1}{2}$;

(b) *S is a non-self-dual non-connected poset of height four.*

5. Conclusions. In this paper we continue study combinatorial aspects of oversupercritical posets. Namely, we describe all the posets that are minimax isomorphic to the oversupercritical poset

$$U = \{1, 2, \dots, 10 \,|\, 1 \prec 2 \prec 3 \prec 4 \prec 5 \prec 6, 7 \prec 8, 9 \prec 10, 7 \prec 10\}.$$

The importance of studying minimax isomorphic posets is determined by the fact that their Tits quadratic forms are $\mathbb{Z}$-equivalent.

We also describe the transitivity coefficients for all posets minimax isomorphic to this oversupercritical poset.

The obtained results (together with the corresponding research methods) can be used in the study of combinatorial aspects of other posets.

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Бондаренко В. М., Стьопочкіна М. В. Про коефіцієнти транзитивності частково впорядкованих множин, що мають надсуперкритичний непримітивний MM-тип.

М. М. Клейнер довів, що ч. в. (частково впорядкована) множина S має скінченний зображувальний тип тоді і лише тоді, коли вона не містить ч. в. підмножин вигляду

$$(1,1,1,1), \quad (2,2,2), \quad (1,3,3), \quad (1,2,5), \quad (\mathrm{N},4).$$

Ці ч. в. множини називаються ч. в. множинами Клейнера і є (з точністю до ізоморфізму) всіма критичними ч. в. множинами щодо скінченності типу (в тому сенсі, що це мінімальні ч. в. множини нескінченного зображувального типу). Пізніше Ю. А. Дрозд довів, що ч. в. множина S має скінченний зображувальний тип тоді і лише тоді, коли квадратична форма

$$q_S(z) =: z_0^2 + \sum_{i\ inS} z_i^2 + \sum_{i<j,i,j\ inS} z_i z_j - z_0 \sum_{i\ inS} z_i,$$

яка називається квадратичною формою Тітса множини S, є слабко додатною (тобто додатною на множині невід'ємних векторів). Отже, ч. в. множини Клейнера є критичними щодо слабкої додатності квадратичної форми Тітса. У 2005 році автори довели що ч. в. множина є критичною щодо додатності квадратичної форми Тітса тоді і лише тоді, коли вона мінімаксно ізоморфна деякій ч. в. множині Клейнера.

Подібну ситуацію маємо для ч. в. множин ручного зображувального типу. Л. А. Назарова довела, що ч. в. множина S є ручною тоді і лише тоді, коли вона не містить ч. в. підмножин вигляду

$$(1,1,1,1,1), \quad (1,1,1,2), \quad (2,2,3), \quad (1,3,4), (1,2,6), \quad (\mathrm{N},5).$$

Ці ч. в. множини є критичними щодо слабкої невід'ємності квадратичної форми Тітса і називаються суперкритичними. У 2009 році автори довели, що ч. в. множина є критичною щодо невід'ємності квадратичної форми Тітса тоді і лише тоді, коли вона мінімаксно ізоморфна деякій суперкритичній ч. в. множині. Перший автор запропонував ввести так звані надсуперкритичні (або 1-надсуперкритичні) ч. в. множини, які відрізняються від суперкритичних ч. в. множин в тій самій мірі, що і останні відрізняються від критичних. Серед цих ч. в. множин є єдина не примітивна, тобто яка не є прямою сумою ланцюгів. У цій статті ми описуємо всі ч. в. множини, які мінімаксно ізоморфні їй, і вивчаємо деякі їхні комбінаторні властивості. Важливість вивчення мінімаксно ізоморфних ч. в. множин визначається тим, що їх квадратичні форми Тітса $\mathbb{Z}$-еквівалентні, а сам мінімаксний ізоморфізм є досить загальною конструктивно визначеною $\mathbb{Z}$-еквівалентністю для квадратичних форм Тітса ч. в. множин.

Ключові слова: зображення, критична та суперкритична ч. в. множина, надсуперкритична ч. в. множина, квадратична форма Тітса, скінченний і ручний зображувальний тип, додатність і слабка додатність, негативність і слабка негативність.

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