УДК 539.3
DOI 10.24144/2616-7700.2022.1(40).94-108

S. Yu. Babich[1], Yu. P. Glukhov[2], V. F. Lazar[3], Yu. Yu. Zhiguts[4]

[1] Institute of Mechanics S.P. Tymoshenko of the National Academy of Sciences of Ukraine,
leading researcher, Professor, Doctor of Technical Sciences,
babich_sy@ukr.net
ORCID: https://orcid.org/0000-0003-2642-9115

[2] Institute of Mechanics S.P. Tymoshenko of the National Academy of Sciences of Ukraine,
Senior Researcher, Candidate of Physical and Mathematical Sciences,
gluchov.uriy@gmail.com
ORCID: https://orcid.org/0000-0002-6328-5993

[3] Mukachevo State University,
Mechanical Engineering, Natural Sciences and Information Technologies, Candidate of Technical Sciences,
vflazar@gmail.com
ORCID: https://orcid.org/0000-0002-2457-571X

[4] Uzhgorod National University,
Professor of the Department of Mechanical Engineering Technology,
Doctor of Engineering
yuzhiguts@gmail.com
ORCID: https://orcid.org/0000-0002-7477-7619

DYNAMICS OF A TWO-LAYER HALF-SPACE WITH INITIAL STRESSES UNDER THE IMPACT OF A MOVING LOAD

In this article, within the framework of the linearized theory of elasticity for bodies with initial stresses, we consider a plane steady-state problem of perturbation of a two-layer half-space with an arbitrary form of elastic potential by a surface load moving at a constant speed with initial stresses. The solution is obtained in a general form for a compressible and incompressible half-space and various contact conditions. Numerical results are given for half-spaces of compressible and incompressible materials, respectively, with an elastic potential of a harmonic type and an elastic potential of the Bartenev-Khazanovich type under rigid and sliding contact conditions.

Keywords: layered half-space, initial stresses, moving load.

1. Introduction. Currently, in the dynamics of elastic bodies with initial (residual) stresses, a number of scientific areas are being developed, of which the following can be noted: studies of the laws of wave propagation in bodies of various shapes (monographs [1,2]); study of the mechanics of moving cracks inhomogeneous materials (for example, [3-6] and a number of other publications) and in the interfaces of materials [7-10]; study of the dynamics of materials under moving loads (for example, [2,11-13] and a number of other publications). A modern analysis of the construction of the main relations of the linearized mechanics of deformed bodies (statics, dynamics, and stability) is presented in publications [14-16] and in a number of others; in this case, in [16], the main attention is paid to the analysis of the features of the construction of constitutive equations for elastic and elastic-plastic materials in the linearized mechanics of deformable bodies. An analysis of the construction of exact solutions to mixed plane problems of linearized mechanics of deformable bodies is presented in [17]; exact solutions are constructed using the apparatus of the theory of functions of complex variables, which are introduced in such a way that the initial (residual) stresses enter into the complex variables. A number of related results in nonlinear and linearized mechanics of deformable bodies are presented in publications [18-20].

In this article, within the framework of formulations [11,12], using the integral Fourier transform, we obtain a solution to the problem in a general form for compressible and incompressible materials and for rigid and sliding contacts between the layer and the base.

2. Statement of the problem. Consider a layer of thickness $2h$, lying on a half-space, the initial stress-strain state of which is determined by the following components of the displacement vector and the generalized stress tensor:

$$u_j^0 = \delta_{ij}\left(\lambda_i + 1\right) x_i; \quad \sigma_{ii}^{*0} \neq 0 \quad (i,\ j = 1,\ 2,\ 3), \tag{1}$$

where λ_i – elongations (λ_i = const) along Lagrangian axes coordinate system x_i, which is overlapping in the natural condition of the Cartesian coordinate system. Along with the Lagrangian coordinates let us bring Cartesian coordinates ξ_i of initial deform condition, connected with coordinates x_i by $\xi_i = \lambda_i x_i$.

To a free boundary layer moving with a constant speed **v** load, independent of the coordinates ξ_3, is attached. Such a load causes plain deformed condition in this layered medium.

For the solution of the task let's seize relations in linearized theory of elasticity for compressible bodies with initial stresses [16]. Assuming that the picture of the deformations is invariant about the time of moving along with a load system (y_1, y_2), where $y_1 = \xi_1 - \mathrm{v}t$; $y_2 = \xi_2$, the equation of the established moving of the semispace by the function $\chi\left(y_1,\ y_2\right)$ can be written as

$$\left(\eta_1^2 \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\right)\left(\eta_2^2 \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\right)\chi^{(j)} = 0, j = 1, 2. \tag{2}$$

Equation roots η_1 and η_2 are formed from the next equation:

$$\eta^4 + 2A\eta^2 + A_1 = 0, \tag{3}$$

$$2A\tilde{\omega}_{2222}\tilde{\omega}_{2112} = \tilde{\omega}_{2222}\left(\tilde{\omega}_{1111} - \tilde{\rho}\mathrm{v}^2\right) + \tilde{\omega}_{2112}\left(\tilde{\omega}_{1221} - \tilde{\rho}\mathrm{v}^2\right) - \left(\tilde{\omega}_{1122} + \tilde{\omega}_{1212}\right)^2; \tag{4}$$
$$2A_1\tilde{\omega}_{2222}\tilde{\omega}_{2112} = \left(\tilde{\omega}_{1111} - \tilde{\rho}\mathrm{v}^2\right)\left(\tilde{\omega}_{1221} - \tilde{\rho}\mathrm{v}^2\right); \quad \tilde{\rho}\lambda_1\lambda_2\lambda_3 = \rho;$$

and in the case of an incompressible material from the relations

$$\begin{gathered} 2A\tilde{q}_{22}^2\tilde{\kappa}_{2112} = \tilde{q}_{11}^2\tilde{\kappa}_{2222} + \tilde{q}_{22}^2\left(\tilde{\kappa}_{1111} - \tilde{\rho}\mathrm{v}^2\right) - 2\tilde{q}_{11}\tilde{q}_{22}\left(\tilde{\kappa}_{1122} + \tilde{\kappa}_{1212}\right); \\ 2A_1\tilde{q}_{22}^2\tilde{\kappa}_{2112} = \tilde{q}_{11}^2\left(\tilde{\kappa}_{1221} - \tilde{\rho}\mathrm{v}^2\right); \quad \tilde{q}_{ij} = \delta_{ij}\lambda_i q_i; \quad \tilde{\rho} = \rho; \end{gathered} \tag{5}$$

In formulas (4) and (5) ρ is the density of the material of the half-space in its natural state.

Let us assume that the motion of the layer can be described by a system of equations from the theory of plates, taking into account the influence of rotational inertia and transverse shear. For a plate under the influence of transverse and tangential surface forces, the corresponding equations are given in [22]. In the coordinate system (y_1, y_2), the equations of plate theory can be written as

$$\begin{gathered} 2h\left(\tfrac{2G_1}{1-\nu_1} - \rho_1\mathrm{v}^2\right)\tfrac{\partial^2 u}{\partial y_1^2} - \tau = P_1; \\ 2h\left(\kappa G_1 - \rho_1\mathrm{v}^2\right)\tfrac{\partial^2 w}{\partial y_1^2} - 2\kappa G_1 h\tfrac{\partial\varphi}{\partial y_1} - q = P_2; \\ \tfrac{2h^2}{3}\left(\tfrac{2G_1}{1-\nu_1} - \rho_1\mathrm{v}^2\delta_0\right)\tfrac{\partial^2\varphi}{\partial y_1^2} + 2\kappa G_1\left(\tfrac{\partial w}{\partial y_1} - \varphi\right) - \tau = 0; \end{gathered} \tag{6}$$

In equations (4) G_1, ν_1, and ρ_1 are, respectively, the shear modulus, Poisson's ratio, and the density of the plate material; u w and are displacements of the middle surface of the plate ($y_2 = 0$), δ_0 is a constant that takes the value 1 or 0 depending on whether the plate rotation inertia is taken into account or neglected when deriving equations (6); φ is the angle of rotation of the plate cross-section; κ is the Timoshenko shift coefficient; q and τ are, respectively, the normal and shear stresses acting on the interface between the plate and the half-space; P_1 and P_2 are the tangential and normal components of the load on the free surface of the plate. The magnitude of the bending moment in the plate is determined by the formula

$$M = \frac{4}{3}\frac{G_1 h^3}{1-\nu_1}\frac{d\varphi}{dy_1}. \tag{7}$$

Let us consider two cases of contact between the plate and the half-space at $y_2 = -h$:

$$\tilde{Q}_{21} = \tau; \quad \tilde{Q}_{22} = q; \quad u_2 = w; \quad u_1 = u + h\varphi; \tag{8}$$

soft contact

$$\tilde{Q}_{21} = 0; \quad \tau = 0; \quad \tilde{Q}_{22} = q; \quad u_2 = w. \tag{9}$$

Thus, the problem is reduced to solving the equations of motion (2) and (6) under boundary conditions (8) or (9).

Using the equations of motion of the plate (6) and conditions (8) and (9), the boundary conditions can be written in the general form

$$\begin{gathered}\delta_1\theta_1\left(\frac{d^2u_1}{dy_1^2} - h\frac{d^2\varphi}{dy_1^2}\right) - \tilde{Q}_{21} = \delta_1 P_1,\\ \theta_3\frac{d^2u_2}{dy_1^2} - 2\kappa h G_1\frac{d\varphi}{dy_1} - \tilde{Q}_{22} = P_2,\\ \theta_2\frac{d^2\varphi}{dy_1^2} + 2\kappa G_1\left(\frac{du_2}{dy_1} - \varphi\right) - \delta_1\tilde{Q}_{21} = 0.\end{gathered} \tag{10}$$

Here we have introduced the following notation

$$\theta_1 = 2h\left(\frac{2G_1}{1-\nu_1} - \rho_1\mathrm{v}^2\right); \quad \theta_2 = \frac{2h^2}{3}\left(\frac{2G_1}{1-\nu_1} - \delta_0\rho_1\mathrm{v}^2\right); \quad \theta_3 = 2h\left(\kappa G_1 - \rho_1\mathrm{v}^2\right).$$

The parameter δ_1 is 1 for hard contact and 0 for soft contact.

The values of the functions $\eta_1^2(\mathrm{v})$ and $\eta_2^2(\mathrm{v})$ determine the form of the equations of motion (2) and, accordingly, the choice of the form for solving the considered equations. The effect of the load movement speed on the value of the roots of equation (3) for a compressible and incompressible half-space is studied in detail in [11,12]. Let us write the solution of the problem in general form for equal and unequal roots of equation (3).

The stresses, displacements, and velocities of displacements in the half-space through functions $\chi^{(j)}$ are determined by the formulas [1]

$$\tilde{Q}_{ij} = \left(\alpha_{ij}^{(12)}\frac{\partial^2}{\partial y_1^2} + \alpha_{ij}^{(22)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial\chi^{(2)}}{\partial y_{2-\delta_{ij}}} + \left(\alpha_{ij}^{(11)}\frac{\partial^2}{\partial y_1^2} + \alpha_{ij}^{(21)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial\chi^{(1)}}{\partial y_{1+\delta_{ij}}}; \quad i,j = 1,2; \tag{11}$$

$$u_i = -\beta_{i1}^{(i)} \frac{\partial^2 \chi^{(i)}}{\partial y_1 \partial y_2} + \left(\beta_{i1}^{(j)} \frac{\partial^2}{\partial y_1^2} + \beta_{i2}^{(j)} \frac{\partial^2}{\partial y_2^2} \right) \chi^{(j)}; \quad i,j = 1,2; \quad i \neq j; \tag{12}$$

$$\dot{u}_i = \mathrm{v} \left[\beta_{i1}^{(i)} \frac{\partial^3 \chi^{(i)}}{\partial y_1^2 \partial y_2} - \left(\beta_{i1}^{(j)} \frac{\partial^2}{\partial y_1^2} + \beta_{i2}^{(j)} \frac{\partial^2}{\partial y_2^2} \right) \frac{\partial \chi^{(j)}}{\partial y_1} \right]; \quad i,j = 1,2; \quad i \neq j; \tag{13}$$

where in the case of a compressible material

$$\alpha_{ii}^{(11)} = \tilde{\omega}_{ii22} \left(\tilde{\omega}_{1111} - \tilde{\rho} \mathrm{v}^2 \right) - \tilde{\omega}_{ii11} \left(\tilde{\omega}_{1212} + \tilde{\omega}_{2211} \right);$$

$$\alpha_{ii}^{(12)} = \tilde{\omega}_{ii11} \left(\tilde{\omega}_{1221} - \tilde{\rho} \mathrm{v}^2 \right); \qquad \alpha_{ii}^{(21)} = \tilde{\omega}_{ii22} \tilde{\omega}_{2112};$$

$$\alpha_{ii}^{(22)} = \tilde{\omega}_{ii11} \tilde{\omega}_{2222} - \tilde{\omega}_{ii22} \left(\tilde{\omega}_{1122} + \tilde{\omega}_{2121} \right);$$

$$\alpha_{ij}^{(11)} = \tilde{\omega}_{ij21} \left(\tilde{\omega}_{1111} - \tilde{\rho} \mathrm{v}^2 \right); \qquad \alpha_{ij}^{(22)} = \tilde{\omega}_{ij12} \tilde{\omega}_{2222};$$

$$\alpha_{ij}^{(12)} = \tilde{\omega}_{ij12} \left(\tilde{\omega}_{1221} - \tilde{\rho} \mathrm{v}^2 \right) - \tilde{\omega}_{ij21} \left(\tilde{\omega}_{1122} + \tilde{\omega}_{2121} \right);$$

$$\alpha_{12}^{(21)} = \tilde{\omega}_{ij21} \tilde{\omega}_{2112} - \tilde{\omega}_{ij12} \left(\tilde{\omega}_{1212} + \tilde{\omega}_{2211} \right);$$

$$\beta_{11}^{(1)} = \beta_{21}^{(2)} = \beta = \tilde{\omega}_{1212} + \tilde{\omega}_{2211};$$

$$\beta_{i2}^{(j)} = \tilde{\omega}_{2jj2}; \quad \beta_{i1}^{(j)} = \tilde{\omega}_{1jj1} - \tilde{\rho} \mathrm{v}^2; \quad i,j = 1,2; \quad i \neq j;$$

and in the case of an incompressible material

$$\alpha_{ii}^{(ii)} = (-1)^i \, \tilde{q}_{jj}^{-1} \tilde{\kappa}_{1212} - \delta_{j2} \tilde{\rho} \mathrm{v}^2 \tilde{q}_{11}^{-1};$$

$$\alpha_{jj}^{(ii)} = \tilde{q}_{jj} \tilde{q}_{ii}^{-2} \left(\tilde{\kappa}_{iiii} - \delta_{j2} \tilde{\rho} \mathrm{v}^2 \right) + \tilde{\kappa}_{jjjj} \tilde{q}_{jj}^{-1} - \tilde{q}_{ii}^{-1} \left(2\tilde{\kappa}_{1122} + \tilde{\kappa}_{1212} \right);$$

$$\alpha_{ij}^{(12)} = -\tilde{q}_{22}^{-1} \tilde{\kappa}_{ij21}; \quad \alpha_{ij}^{(22)} = \tilde{q}_{11}^{-1} \tilde{\kappa}_{ij12}; \quad \alpha_{ij}^{(11)} = \tilde{q}_{22}^{-1} \tilde{\kappa}_{ij21};$$

$$\alpha_{ij}^{(21)} = -\tilde{q}_{11}^{-1} \tilde{\kappa}_{ij12}; \quad i,j = 1,2; \quad i \neq j;$$

$$\alpha_{22}^{(12)} = \tilde{q}_{22}^{-1} \left(\tilde{\kappa}_{1221} - \tilde{\rho} \mathrm{v}^2 \right); \quad \alpha_{11}^{(12)} = \tilde{q}_{11} \tilde{q}_{22}^{-1} \alpha_{22}^{(12)};$$

$$\alpha_{11}^{(21)} = \tilde{q}_{11}^{-1} \tilde{\kappa}_{2112}; \quad \alpha_{22}^{(21)} = \tilde{q}_{22} \tilde{q}_{11}^{-1} \alpha_{11}^{(21)};$$

$$\beta_{11}^{(1)} = \beta_{12}^{(2)} = \beta = \tilde{q}_{11}^{-1}; \quad \beta_{21}^{(2)} = \beta_{21}^{(1)} = \tilde{q}_{22}^{-1}; \quad \beta_{11}^{(2)} = \beta_{22}^{(1)} = 0;$$

Taking into account (11) and (12), boundary conditions (10) can be written as

$$\begin{gathered}
\left[\delta_1 \theta_1 \frac{\partial^2}{\partial y_1^2} \left(\beta_{11}^{(2)} \frac{\partial^2}{\partial y_1^2} + \beta_{12}^{(2)} \frac{\partial^2}{\partial y_2^2} \right) - \frac{\partial}{\partial y_2} \left(\alpha_{21}^{(12)} \frac{\partial^2}{\partial y_1^2} + \alpha_{21}^{(22)} \frac{\partial^2}{\partial y_2^2} \right) \right] \chi^{(2)} - \\
- \left[\delta_1 \theta_1 \beta_{11}^{(1)} \frac{\partial^3}{\partial y_1^2 \partial y_2} + \left(\alpha_{21}^{(11)} \frac{\partial^2}{\partial y_1^2} + \alpha_{21}^{(21)} \frac{\partial^2}{\partial y_2^2} \right) \right] \frac{\partial \chi^{(1)}}{\partial y_1} - \delta_1 \theta_1 h \frac{\partial^2 \varphi}{\partial y_1^2} = \delta_1 P_1; \\
-2\kappa h G_1 \frac{\partial \varphi}{\partial y_1} - \left[\theta_3 \beta_{21}^{(2)} \frac{\partial^3}{\partial y_1^2 \partial y_2} + \left(\alpha_{22}^{(12)} \frac{\partial^2}{\partial y_1^2} + \alpha_{22}^{(22)} \frac{\partial^2}{\partial y_2^2} \right) \right] \frac{\partial \chi^{(2)}}{\partial y_1} + \\
+ \left[\theta_3 \frac{\partial^2}{\partial y_1^2} \left(\beta_{21}^{(1)} \frac{\partial^2}{\partial y_1^2} + \beta_{22}^{(1)} \frac{\partial^2}{\partial y_2^2} \right) - \frac{\partial}{\partial y_2} \left(\alpha_{22}^{(11)} \frac{\partial^2}{\partial y_1^2} + \alpha_{22}^{(21)} \frac{\partial^2}{\partial y_2^2} \right) \right] \chi^{(1)} = P_2; \qquad (14) \\
\theta_2 \frac{\partial^2 \varphi}{\partial y_1^2} - 2\kappa G_1 \varphi - \left[\left(2\kappa G_1 \beta_{21}^{(2)} + \delta_1 \alpha_{21}^{(12)} \right) \frac{\partial^2}{\partial y_1^2} + \delta_1 \alpha_{21}^{(22)} \frac{\partial^2}{\partial y_2^2} \right] \frac{\partial \chi^{(2)}}{\partial y_2} +
\end{gathered}$$

$$+\left[\left(2\kappa G_1\beta_{21}^{(1)}-\delta_1\alpha_{21}^{(11)}\right)\frac{\partial^2}{\partial y_1^2}+\left(2\kappa G_1\beta_{22}^{(1)}-\delta_1\alpha_{21}^{(21)}\right)\frac{\partial^2}{\partial y_2^2}\right]\frac{\partial\chi^{(1)}}{\partial y_1}=0;$$

Thus, the problem of the steady motion of a two-layer compressible half-space under the action of a moving load is reduced to finding the functions $\chi^{(j)}$ and φ from the boundary conditions (14).

3. Solution of the problem in the field of images.. We find the solution of the problem using the integral Fourier transform with respect to the variable and the corresponding inversion formula. Applying the Fourier transform to equations (2), we obtain

$$\left(\frac{d^2}{dy_2^2}-k^2\eta_1^2\right)\left(\frac{d^2}{dy_2^2}-k^2\eta_2^2\right)\chi^{(j)F}=0;\quad j=1,2. \tag{15}$$

Let us define the solution of the problem in a general form for the cases of unequal and equal roots, for various conditions of conjugation of the layer and half-space, and for any speed of the load (subsonic, transonic, and supersonic).

Boundary conditions (14) in the space of Fourier images have the form

$$\begin{gathered}\left(-\alpha_{21}^{(22)}\frac{d^3}{dy_2^3}-k^2\delta_1\theta_1\beta_{12}^{(2)}\frac{d^2}{dy_2^2}+k^2\alpha_{21}^{(12)}\frac{d}{dy_2}+k^4\delta_1\theta_1\beta_{11}^{(2)}\right)\chi^{(2)F}-\\ -ik\left(\alpha_{21}^{(21)}\frac{d^2}{dy_2^2}-k^2\delta_1\theta_1\beta_{11}^{(1)}\frac{d}{dy_2}-k^2\alpha_{21}^{(11)}\right)\chi^{(1)F}+k^2\delta_1\theta_1 h\varphi^F=\delta_1P_1^F;\\ -2ik\kappa hG_1\varphi^F+ik\left(-\alpha_{22}^{(22)}\frac{d^2}{dy_2^2}+k^2\theta_3\beta_{21}^{(2)}\frac{d}{dy_2}+k^2\alpha_{22}^{(12)}\right)\chi^{(2)F}-\\ -\left(\alpha_{22}^{(21)}\frac{d^3}{dy_2^3}+k^2\theta_3\beta_{22}^{(1)}\frac{d^2}{dy_2^2}-k^2\alpha_{22}^{(11)}\frac{d}{dy_2}-k^4\theta_3\beta_{21}^{(1)}\right)\chi^{(1)F}=P_2^F;\\ \left(k^2\theta_2+2\kappa G_1\right)\varphi^F-\left[k^2\left(2\kappa G_1\beta_{21}^{(2)}+\delta_1\alpha_{21}^{(12)}\right)-\delta_1\alpha_{21}^{(22)}\frac{d^2}{dy_2^2}\right]\frac{d\chi^{(2)F}}{dy_2}+\\ +ik\left[k^2\left(2\kappa G_1\beta_{21}^{(1)}-\delta_1\alpha_{21}^{(11)}\right)-\left(2\kappa G_1\beta_{22}^{(1)}-\delta_1\alpha_{21}^{(21)}\right)\frac{d^2}{dy_2^2}\right]\chi^{(1)F}=0;\end{gathered} \tag{16}$$

The solution of the transformed equation (15), taking into account damping at infinity, will be sought in the form

$$\begin{gathered}\chi^{F(j)}=\left[1-\delta_{j2}(1-\delta_{\eta_1\eta_2})\right]\times\\ \times\left\{C_1^{(j)}\mathrm{e}^{k_1k\eta_1(y_2+h)}+\left[\delta_{\eta_1\eta_2}(y_2+h)+1-\delta_{\eta_1\eta_2}\right]C_2^{(j)}e^{k_2k\eta_2(y_2+h)}\right\};\end{gathered} \tag{17}$$

where $C_m^{(j)}\quad (j,m=1,2)$ are constants of integration,

$$\gamma_j=k_j\eta_j;\quad j=1,2;\quad \delta_{\eta_1\eta_2}=\left\{\begin{array}{ll}0, & \eta_1\neq\eta_2\\ 1, & \eta_1=\eta_2\end{array}\right.;\quad \delta_{j2}=\left\{\begin{array}{ll}0, & j=1\\ 1, & j=2\end{array}\right..$$

Let us introduce constants of integration

$$C_1^{(1)}=iC_1;\quad C_2^{(1)}=iC_2;\quad C_1^{(2)}=C_1;\quad C_2^{(2)}=C_2; \tag{18}$$

Substituting (17) and (18) into (16), we obtain a system of algebraic equations for the unknowns $C_1,\ C_2$ and φ^F

$$k\left(a_{11}^{(1)}+ka_{11}^{(2)}\right)C_1+\left(a_{12}^{(1)}+ka_{12}^{(2)}+k^2a_{12}^{(3)}\right)C_2+a_{13}\varphi^F=k^{-2}\delta_1P_1^F;$$
$$k^2\left(a_{21}^{(1)}+ka_{21}^{(2)}\right)C_1+k\left(a_{22}^{(1)}+ka_{22}^{(2)}+k^2a_{22}^{(3)}\right)C_2+a_{23}\varphi^F=-ik^{-1}P_2^F; \tag{19}$$
$$k^3a_{31}C_1+k^2\left(a_{32}^{(1)}+ka_{32}^{(2)}\right)C_2+\left(a_{33}^{(1)}+k^2a_{33}^{(2)}\right)\varphi^F=0;$$

where

$$a_{11}^{(1)}=-\gamma_{21}^{(11)}+\delta_{\eta_1\eta_2}\gamma_1\gamma_{21}^{(21)};\quad a_{11}^{(2)}=\delta_1\theta_1\left(\delta_{\eta_1\eta_2}\theta_1^{(21)}-\beta_{11}^{(1)}\gamma_1\right);$$
$$a_{12}^{(1)}=\delta_{\eta_1\eta_2}\left[\gamma_{21}^{(22)}+2\gamma_2\left(\alpha_{21}^{(21)}-\alpha_{21}^{(22)}\gamma_2\right)\right];$$
$$a_{12}^{(2)}=-\left[\delta_1\delta_{\eta_1\eta_2}\theta_1\left(\beta_{11}^{(1)}+2\beta_{12}^{(2)}\gamma_2\right)+\left(1-\delta_{\eta_1\eta_2}\right)\gamma_{21}^{(12)}\right];$$
$$a_{12}^{(3)}=-\delta_1\theta_1\beta_{11}^{(1)}\gamma_2\left(1-\delta_{\eta_1\eta_2}\right);\quad a_{13}=\delta_1\theta_1h;$$
$$a_{21}^{(1)}=\gamma_1\gamma_{22}^{(11)}+\delta_{\eta_1\eta_2}\gamma_{22}^{(21)};\quad a_{21}^{(2)}=\theta_3\left(\theta_2^{(11)}+\delta_{\eta_1\eta_2}\beta_{21}^{(2)}\gamma_1\right);$$
$$a_{22}^{(1)}=\delta_{\eta_1\eta_2}\left[\gamma_{22}^{(12)}-2\gamma_2\left(\alpha_{22}^{(22)}+\alpha_{22}^{(21)}\gamma_2\right)\right];$$
$$a_{22}^{(2)}=\delta_{\eta_1\eta_2}\theta_3\left(\beta_{21}^{(2)}-2\beta_{22}^{(1)}\gamma_2\right)+\left(1-\delta_{\eta_1\eta_2}\right)\gamma_2\gamma_{22}^{(12)};$$
$$a_{22}^{(3)}=\theta_3\theta_2^{(12)}\left(1-\delta_{\eta_1\eta_2}\right);\quad a_{23}=-2\kappa hG_1;$$
$$a_{31}=2\kappa G_1\left(\theta_2^{(11)}+\delta_{\eta_1\eta_2}\gamma_1\beta_{21}^{(2)}\right)+\delta_1\left(\delta_{\eta_1\eta_2}\gamma_1\gamma_{21}^{(21)}-\gamma_{21}^{(11)}\right);$$
$$a_{32}^{(2)}=\left(1-\delta_{\eta_1\eta_2}\right)\left(2\kappa G_1\theta_2^{(12)}-\delta_1\gamma_{21}^{(12)}\right);$$
$$a_{32}^{(1)}=\delta_{\eta_1\eta_2}\left\{2\kappa G_1\left(\beta_{21}^{(2)}-2\gamma_2\beta_{22}^{(1)}\right)+\delta_1\left[\gamma_{21}^{(22)}+2\gamma_2\left(\alpha_{21}^{(21)}-\alpha_{21}^{(22)}\gamma_2\right)\right]\right\};$$
$$a_{33}^{(1)}=-2\kappa G_1;\quad a_{33}^{(2)}=-\theta_2;$$
$$\theta_m^{(kj)}=\beta_{m1}^{(k)}-\beta_{m2}^{(k)}\gamma_j^2;\quad \gamma_{mk}^{(nj)}=\alpha_{mk}^{(1n)}-\alpha_{mk}^{(2n)}\gamma_j^2;\quad j,k,m=1,2;$$

The solution of system (19) can be written as follows

$$C_j=\frac{\delta_1P_1^FU_1^{(j)}+iP_2^FU_2^{(j)}}{\Delta(k)};\quad j=1,2;\quad \varphi^F=\frac{\delta_1P_1^FU_1+iP_2^FU_2}{\Delta(k)}; \tag{20}$$

where

$$\Delta(k)=k^2\left(b_0+kb_1+k^2b_2+k^3b_3+k^4b_4+k^5b_5\right);$$
$$U_j^{(1)}=k^{-1}\left(b_{10}^{(j)}+kb_{11}^{(j)}+k^2b_{12}^{(j)}+k^3b_{13}^{(j)}+k^4b_{14}^{(j)}\right);$$
$$U_j^{(2)}=-\left(b_{20}^{(j)}+kb_{21}^{(j)}+k^2b_{22}^{(j)}+k^3b_{23}^{(j)}\right);\quad U_j=k^2\left(b_{30}^{(j)}+kb_{31}^{(j)}+k^2b_{32}^{(j)}\right);\quad j=1,2;$$
$$b_0=a_{33}^{(1)}\left(a_{11}^{(1)}a_{22}^{(1)}-a_{12}^{(1)}a_{21}^{(1)}\right);$$

$$b_1 = a_{33}^{(1)} \left(a_{11}^{(2)} a_{22}^{(1)} + a_{22}^{(2)} a_{11}^{(1)} - a_{12}^{(1)} a_{21}^{(2)} - a_{12}^{(2)} a_{21}^{(1)} \right) + a_{23} \left(a_{31} a_{12}^{(1)} - a_{11}^{(1)} a_{32}^{(1)} \right);$$

$$b_2 = a_{33}^{(1)} \left(a_{22}^{(2)} a_{11}^{(2)} + a_{22}^{(3)} a_{11}^{(1)} - a_{12}^{(2)} a_{21}^{(2)} - a_{12}^{(3)} a_{21}^{(1)} \right) +$$

$$+ a_{23} \left(a_{31} a_{12}^{(2)} - a_{11}^{(1)} a_{32}^{(2)} - a_{11}^{(2)} a_{32}^{(1)} \right) + a_{33}^{(2)} \left(a_{11}^{(1)} a_{22}^{(1)} - a_{12}^{(1)} a_{21}^{(1)} \right) + a_{13} \left(a_{21}^{(1)} a_{32}^{(1)} - a_{31} a_{22}^{(1)} \right);$$

$$b_3 = a_{33}^{(2)} \left(a_{11}^{(2)} a_{22}^{(1)} + a_{22}^{(2)} a_{11}^{(1)} - a_{12}^{(1)} a_{21}^{(2)} - a_{12}^{(2)} a_{21}^{(1)} \right) +$$

$$+ a_{13} \left(a_{21}^{(1)} a_{32}^{(2)} + a_{21}^{(2)} a_{32}^{(1)} - a_{31} a_{22}^{(2)} \right) + a_{23} \left(a_{31} a_{12}^{(3)} - a_{11}^{(2)} a_{32}^{(2)} \right) + a_{33}^{(1)} \left(a_{22}^{(3)} a_{11}^{(2)} - a_{12}^{(3)} a_{21}^{(2)} \right);$$

$$b_4 = a_{33}^{(2)} \left(a_{11}^{(2)} a_{22}^{(2)} + a_{11}^{(1)} a_{22}^{(3)} - a_{12}^{(2)} a_{21}^{(2)} - a_{12}^{(3)} a_{21}^{(1)} \right) + a_{13} \left(a_{21}^{(2)} a_{32}^{(2)} - a_{22}^{(3)} a_{31} \right);$$

$$b_5 = a_{33}^{(2)} \left(a_{22}^{(3)} a_{11}^{(2)} - a_{12}^{(3)} a_{21}^{(2)} \right);$$

$$b_{10}^{(1)} = a_{22}^{(1)} a_{33}^{(1)}; \quad b_{11}^{(1)} = a_{22}^{(2)} a_{33}^{(1)} - a_{23} a_{32}^{(1)}; \quad b_{12}^{(1)} = a_{22}^{(3)} a_{33}^{(1)} + a_{33}^{(2)} a_{22}^{(1)} - a_{23} a_{32}^{(2)};$$

$$b_{13}^{(1)} = a_{33}^{(2)} a_{22}^{(2)}; \quad b_{14}^{(1)} = a_{33}^{(2)} a_{22}^{(3)};$$

$$b_{10}^{(2)} = a_{12}^{(1)} a_{33}^{(1)}; \quad b_{11}^{(2)} = a_{12}^{(2)} a_{33}^{(1)}; \quad b_{12}^{(2)} = a_{12}^{(3)} a_{33}^{(1)} + a_{33}^{(2)} a_{12}^{(1)} - a_{13} a_{32}^{(1)};$$

$$b_{13}^{(2)} = a_{33}^{(2)} a_{12}^{(2)} - a_{13} a_{32}^{(2)}; \quad b_{14}^{(2)} = a_{33}^{(2)} a_{12}^{(3)};$$

$$b_{20}^{(1)} = a_{21}^{(1)} a_{33}^{(1)}; \quad b_{21}^{(1)} = a_{21}^{(2)} a_{33}^{(1)} - a_{23} a_{31}; \quad b_{22}^{(1)} = a_{21}^{(1)} a_{33}^{(2)}; \quad b_{23}^{(1)} = a_{21}^{(2)} a_{33}^{(2)};$$

$$b_{20}^{(2)} = a_{11}^{(1)} a_{33}^{(1)}; \quad b_{21}^{(2)} = a_{11}^{(2)} a_{33}^{(1)}; \quad b_{22}^{(2)} = a_{11}^{(1)} a_{33}^{(2)} - a_{13} a_{31}; \quad b_{23}^{(2)} = a_{11}^{(2)} a_{33}^{(2)};$$

$$b_{30}^{(1)} = a_{21}^{(1)} a_{32}^{(1)} - a_{22}^{(1)} a_{31}; \quad b_{31}^{(1)} = a_{21}^{(1)} a_{32}^{(2)} + a_{21}^{(2)} a_{32}^{(1)} - a_{22}^{(2)} a_{31}; \quad b_{32}^{(1)} = a_{21}^{(2)} a_{32}^{(2)} - a_{22}^{(3)} a_{31};$$

$$b_{30}^{(2)} = a_{11}^{(1)} a_{32}^{(1)} - a_{12}^{(1)} a_{31}; \quad b_{31}^{(2)} = a_{11}^{(1)} a_{32}^{(2)} + a_{11}^{(2)} a_{32}^{(1)} - a_{12}^{(2)} a_{31}; \quad b_{32}^{(2)} = a_{11}^{(2)} a_{32}^{(2)} - a_{12}^{(3)} a_{31};$$

We apply the Fourier transform to formulas (7), (11) and (13)

$$\begin{gathered} \tilde{Q}_{jm}^F = \left(-k^2 \alpha_{jm}^{(12-\delta_{jm})} + \alpha_{jm}^{(22-\delta_{jm})} \frac{d^2}{dy_2^2} \right) \frac{d\chi^{(2-\delta_{jm})F}}{dy_2} + \\ + ik \left(-k^2 \alpha_{jm}^{(11+\delta_{jm})} + \alpha_{jm}^{(21+\delta_{jm})} \frac{d^2}{dy_2^2} \right) \chi^{(1+\delta_{jm})F}; \quad j, m = 1, 2; \end{gathered} \tag{21}$$

$$\dot{u}_j^F = -ik\text{v} \left(-k^2 \beta_{j1}^{(m)} + \beta_{j2}^{(m)} \frac{d^2}{dy_2^2} \right) \chi^{(m)F} - k^2 \text{v} \beta_{j1}^{(j)} \frac{d\chi^{(j)F}}{dy_2}; \quad j, m = 1, 2; \quad i \neq m;$$

$$M^F = \frac{4}{3} \frac{ikG_1 h^3}{1 - \nu_1} \varphi^F;$$

Taking into account (17), (18), and (20), expressions (21) can be represented as

$$\begin{gathered} \tilde{Q}_{mj}^F = (-i)^{\delta_{mj}} k^2 \Delta^{-1}(k) \left(\delta_1 P_1^F \Gamma_{mj}^{(1)} + i P_2^F \Gamma_{mj}^{(2)} \right); \\ \dot{u}_j^F = i^{2-j} \text{v} k^2 \Delta^{-1}(k) \left(\delta_1 P_1^F \Gamma_2^{(1)} + i P_2^F \Gamma_2^{(2)} \right); \quad m, j = 1, 2; \\ M^F = k \Delta^{-1}(k) \left(i \delta_1 P_1^F \Gamma_\varphi^{(1)} - P_2^F \Gamma_\varphi^{(2)} \right); \end{gathered} \tag{22}$$

where

$$\Gamma_{mm}^{(j)} = k\left(\gamma_1\gamma_{mm}^{(11)} + \delta_{\eta_1\eta_2}\gamma_{mm}^{(21)}\right)U_j^{(1)}e^{k\gamma_1(y_2+h)} - \left\{\delta_{\eta_1\eta_2}\left[2\gamma_2\left(\gamma_2\alpha_{mm}^{(21)} + \alpha_{mm}^{(22)}\right) - \gamma_{mm}^{(12)}\right] - \right.$$

$$\left. -k\left\{\delta_{\eta_1\eta_2}\left(y_2+h\right)\left(\gamma_2\gamma_{mm}^{(12)} + \gamma_{mm}^{(22)}\right) + \left(1-\delta_{\eta_1\eta_2}\right)\gamma_2\gamma_{mm}^{(12)}\right\}\right\}U_j^{(2)}e^{k\gamma_2(y_2+h)};$$

$$\Gamma_{mn}^{(j)} = k\left(\gamma_{mn}^{(11)} - \delta_{\eta_1\eta_2}\gamma_1\gamma_{mn}^{(21)}\right)U_j^{(1)}e^{k\gamma_1(y_2+h)} + \left\{\delta_{\eta_1\eta_2}\left[2\gamma_2\left(\gamma_2\alpha_{mn}^{(22)} - \alpha_{21}^{(21)}\right) - \gamma_{mn}^{(22)}\right] + \right.$$

$$\left. +k\left[\delta_{\eta_1\eta_2}\left(y_2+h\right)\left(\gamma_{mn}^{(12)} - \gamma_2\gamma_{mn}^{(22)}\right) + \left(1-\delta_{\eta_1\eta_2}\right)\gamma_{mn}^{(12)}\right]\right\}U_j^{(2)}e^{k\gamma_2(y_2+h)};$$

$$\Gamma_{\varphi}^{(j)} = \frac{4}{3}\frac{G_1h^3U_j}{(1-\nu_1)}.$$

Thus, the solution of the problem of the steady motion of a two-layer elastic half-space with initial stresses under the influence of a moving load in the region of Fourier images has the form (22).

From (22) it follows that the value of the quantities characterizing the stress-strain state of a two-layer elastic half-space increases without limit at $\Delta(k) \to 0$. Under the condition that real positive multiple roots of the equation $\Delta(k) = 0$ exist, resonance is possible [22].

The results of studying the function $\Delta(k)$ for a compressible and incompressible half-space and various cases of conjugation of a plate and a half-space are given in [11,12].

It follows from the results obtained in [11,12] that the number of critical velocities of the load movement significantly depends on the initial stresses in the half-space, the mechanical characteristics of the plate and the half-space, and the conditions of their contact. The effect of initial stresses on the values of critical velocities is more significant for relatively soft plates and for non-rigid contacts. The value of the lowest critical speed for a non-rigid contact is always less than for a hard one.

4. Numerical studies. In order to pass in formulas (22) to the originals, one should use the inverse Fourier transform.

It follows from the results obtained in [11,12] that the calculation of the inversion integrals essentially depends on the speed of the load. Depending on the velocity v, the denominator $\Delta(k)$ in the inversion integrals may or may not have real positive roots. If no root lies on the real axis, then the inversion integrals have no singularities and can be calculated directly using tables. In the presence of unequal real positive roots of the denominator $\Delta(k)$, the integrals along the integration contour from to can be replaced by the sum of the principal value of the integral $-\infty + i\gamma$ and $+\infty + i\gamma$ the sum of all residues multiplied by $(-i\pi)$ [21]. In the case of the existence of a double positive root, the inversion integrals do not exist even in the Cauchy sense, i.e. resonance appears.

Since it was assumed in the formulation of the problem that the perturbations caused by the moving load are very small, the resonant region was excluded from consideration.

Figures 1-4 show how the initial stresses in the base affect the characteristics of the stress-strain state in a two-layer half-space at different speeds of the load (subsonic, transonic, and supersonic) and the conditions of contact between the plate and the half-space.

The following designations are used in the figures: c_{11} and c_{12} are the velocities of propagation in the direction of the axis Oy_1, respectively, of longitudinal and transverse polarized waves in an unlimited compressible body with initial stresses, c_1 and c_2 are the velocities of propagation of transverse waves in the direction of the axes Oy_1 and Oy_2 in an unlimited incompressible body with initial stresses, c_s is the speed of movement of shear waves in the layer, v^* is the critical speed of the load [11,12].

As an example, a compressible half-space with an elastic potential of harmonic type and an incompressible half-space with the Bartenev-Khazanovich potential are considered [1]. It was assumed that the initial deformed state is flat and there is no surface load.

The calculation results are given for a concentrated linear load, the normal and tangential components of which are determined by the formulas

$$P_1 = P\delta(y_1)\cos\alpha; \quad P_2 = P\delta(y_1)\sin\alpha; \quad P = G_1;$$

where α is the angle of inclination of the load to the axis Oy_1.

5. Conclusions and prospects for further research. An analysis of the results shows that the presence of initial stresses has a significant effect on the distribution of stresses and displacement velocities in the half-space and the bending moment in the plate. This effect is different depending on the position of the considered point of the layered body relative to the point of application of the load.

The values of the parameters of the stress-strain state at a particular point of the layered body depend on the initial stresses, its coordinates, and contact conditions.

For subcritical speeds of movement of the load with rigid contact of stress, the speed of movement in the half-space and the bending moment in the plate is less than with non-rigid contact. At the same time, in the studied range of values, the growth rate of the amplitude of the studied quantities during compression is greater than during tension. Attenuation with distance from the point of application of the load in compression is slower than in tension.

The influence of the initial stresses increases significantly with the increase in the speed of the load. This is especially true during pre-compression. With rigid contact, the influence of velocity and initial stresses is less significant than with non-rigid contact.

Accounting for rotational inertia within the considered velocities of the surface load and values λ_1 in the case of rigid contact introduces an insignificant correction (less than 2.6%), but in the case of non-rigid contact, the difference in the results will be very large (up to 30%). It is especially necessary to take into account the inertia of rotation at $\lambda_1 < 1$ and high speeds of the load.

It can be seen that as the velocity increases, the symmetry is more and more violated, and the direct wave decays much faster and is practically absent in the supersonic case. However, it does not completely disappear. This is apparently explained by the layering of the medium.

In the case of rigid contact, the direct wave decays much faster than in a non-rigid contact.

Harmonic potential

$(\rho/\rho_1 = 0{,}5;\ \ \mu/G_1 = 0{,}5;\ \ \kappa = 0{,}845;\ \ \nu = 0{,}3;\ \ \nu_1 = 0{,}25;\ \ \alpha = \pi/2;\ \ \delta_0 = 1)$

$\mathrm{v} < \mathrm{v}^* < c_{12};\ \ \mathrm{v}^2 = 0{,}1c_s^2 \qquad c_{12} < \mathrm{v} < c_{11};\ \ \mathrm{v}^2 = 2c_s^2 \qquad \mathrm{v} > c_{11};\ \ \mathrm{v}^2 = 6c_s^2$

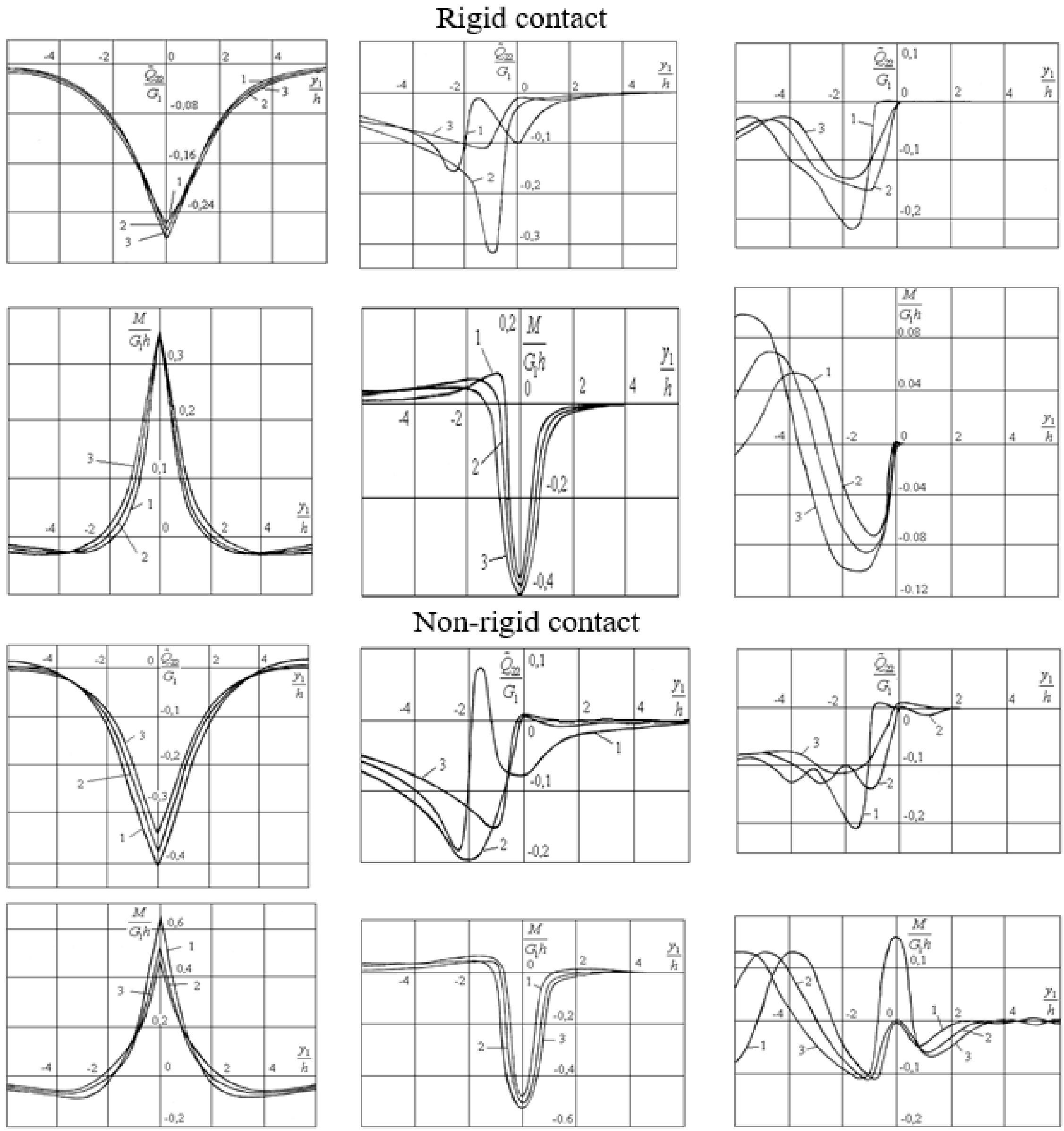


Figure 1. Distribution of stresses and displacement velocities in the half-space at depth $y_2 = -2h/\lambda_2$ and the bending moment in the plate at depth $y_2 = -h/2$ (curve 1 corresponds to $\lambda_1 = 0{,}8$; curve 2 – $\lambda_1 = 1$; curve 3 – $\lambda_1 = 1{,}2$)

Elastic potential of the Bartenev-Khazanovich type
($\kappa = 0{,}845$; $\mu/G_1 = 0{,}5$; $\rho/\rho_1 = 0{,}5$; $\nu_1 = 0{,}25$; $\alpha = \pi/2$; $\delta_0 = 1$)

$\mathrm{v} < \mathrm{v}^* < c_1$; $\mathrm{v}^2 = 0{,}1c_s^2$ $\qquad$ $\mathrm{v} > c_1$; $\mathrm{v}^2 = 2c_s^2$

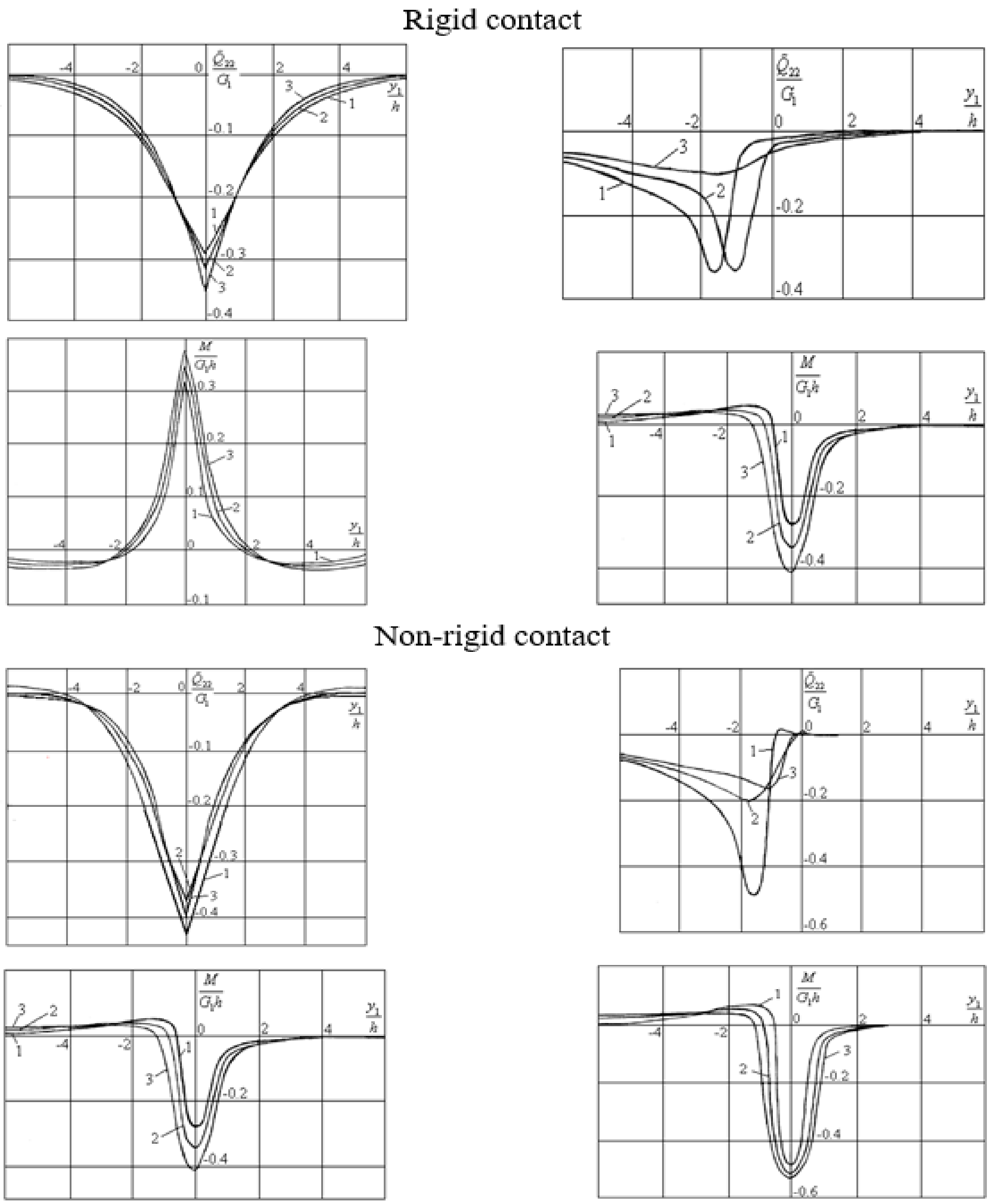
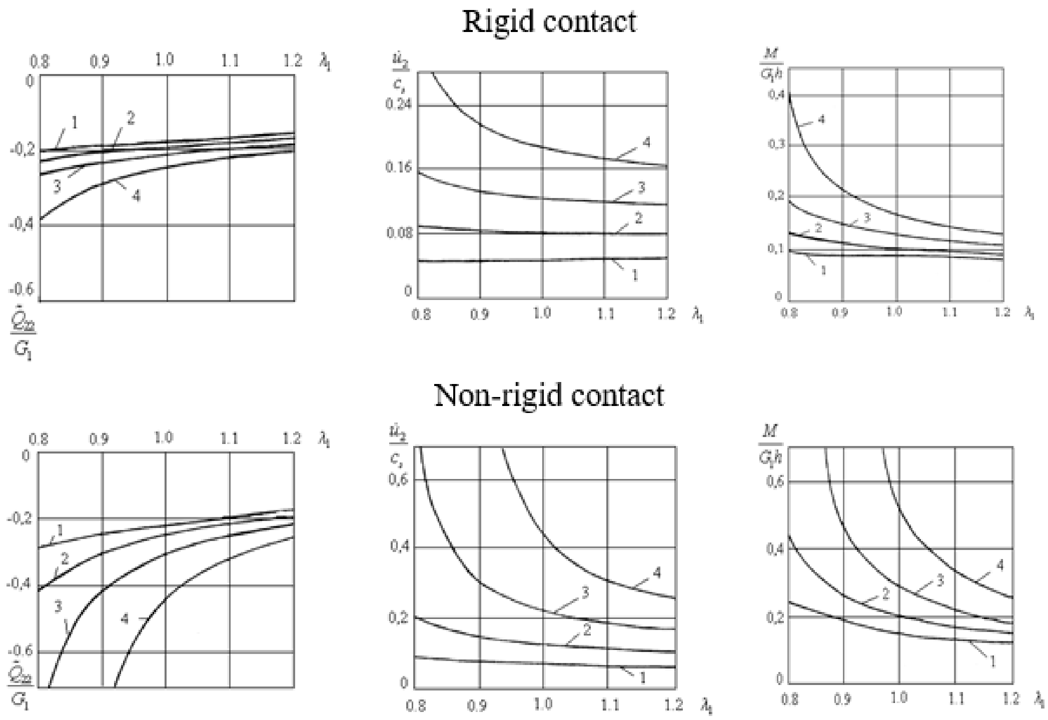


Figure 2. Distribution of stresses and displacement velocities in the half-space at depth $y_2 = -2h/\lambda_2$ and the bending moment in the plate at depth $y_2 = -h/2$ (curve 1 corresponds to $\lambda_1 = 0{,}8$; curve 2 – $\lambda_1 = 1$; curve 3 – $\lambda_1 = 1,2$)

Harmonic potential



Elastic potential of the Bartenev-Khazanovich type

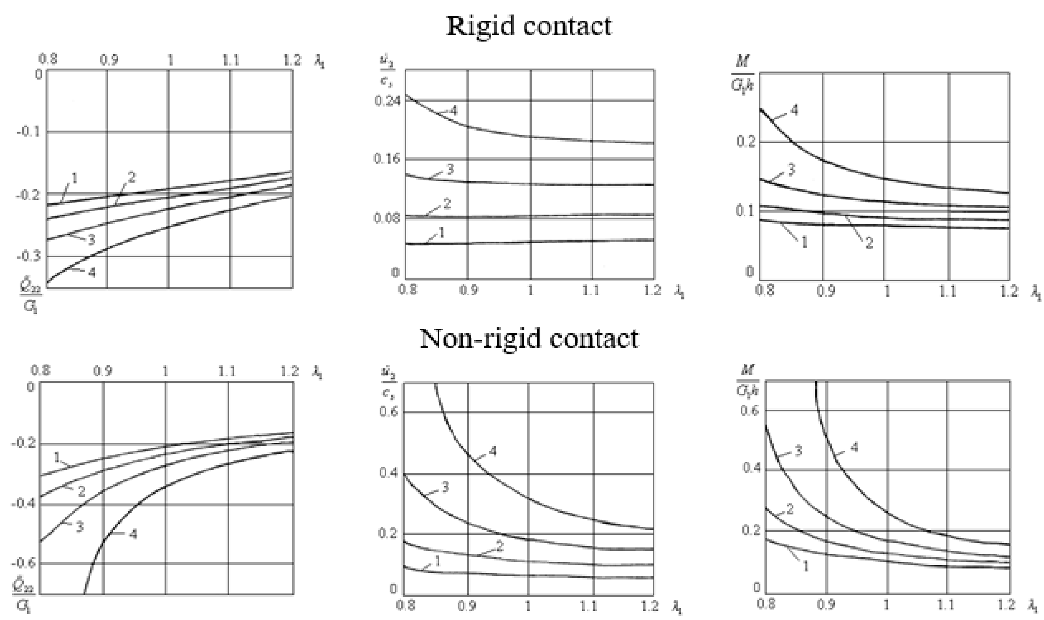


Figure 3. Dependence of the values characterizing the stress-strain state of the underlying half-space on the initial stresses at different subcritical speeds of the load at the point $y_1 = -\lambda_1 h$, $y_2 = -2h/\lambda_2$ (curve 1 corresponds to $\mathrm{v}^2 = 0{,}1c_s^2$, curve 2 – $\mathrm{v}^2 = 0{,}2c_s^2$, curve 3 – $\mathrm{v}^2 = 0{,}3c_s^2$, curve 4 – $\mathrm{v}^2 = 0{,}4c_s^2$)

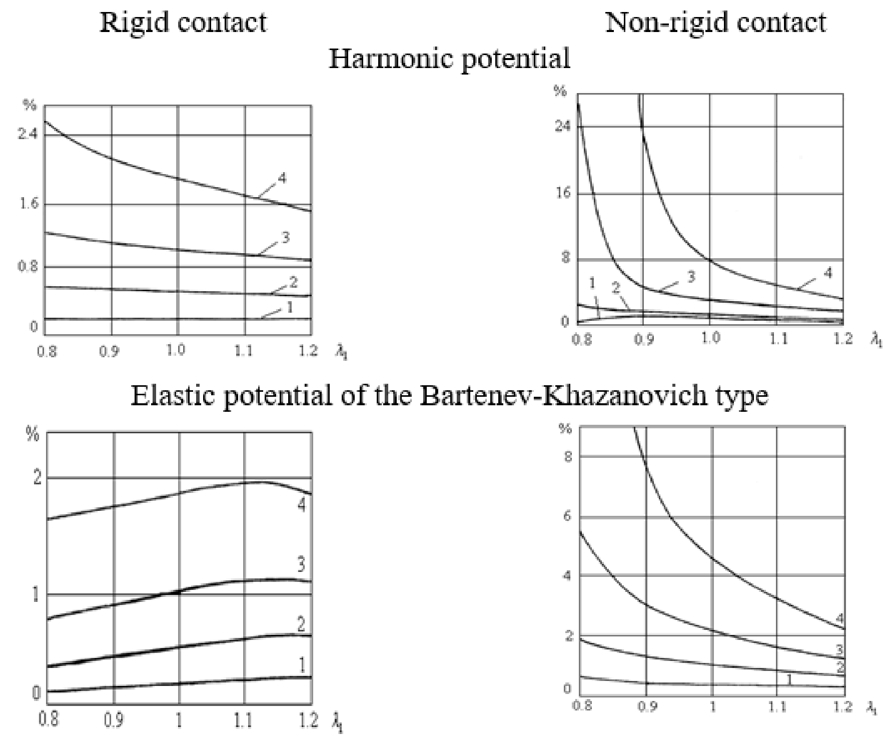


Figure 4. Influence of taking into account the inertia of rotation at different speeds of movement of the load and initial deformations on the value of the stress component $\tilde{Q}_{22}$ at the point $y_1 = -\lambda_1 h$; $y_2 = -2h/\lambda_2$ (curve 1 corresponds to $v^2 = 0{,}1c_s^2$, curve 2 to $v^2 = 0{,}2c_s^2$, curve 3 to $v^2 = 0{,}3c_s^2$, curve 4 to $v^2 = 0{,}4c_s^2$)

References

1. Guz, A. N. (2004). Elastic waves in bodies with initial (residual) stresses. *Kiev*: *Publisher "A.S.K"*.
2. Guz, A. N., Babich, S. Yu., & Glukhov, Yu. P. (2007) Statics and dynamics of the elastic grounds with initial (residual) stresses. *Kremenchug*: *Publisher "Kremenchug"*.
3. Guz, A. N. (1998). Dynamic problems of the mechanics of the brittle fracture of materials with initial stresses for moving cracks. 1. Problem statement and general relationships. *Int. Appl. Mech.,* 34(12), 1175–1186.
4. Guz, A. N. (1999). Dynamic problems of the mechanics of the brittle fracture of materials with initial stresses for moving cracks. 2. Craks of normal separation (Mode I). *Int. Appl. Mech.*, 35(1), 1–12.
5. Guz, A. N. (1999). Dynamic problems of the mechanics of the brittle fracture of materials with initial stresses for moving cracks. 3. Transverse-Shear (Mode II) and Longitudinal-Shear (Mode III). *Int. Appl. Mech.,* 35(2), P. 109–119.
6. Guz, A. N. (1999). Dynamic problems of the mechanics of the brittle fracture of materials with initial stresses for moving cracks. 4. Wedge problems. *Int. Appl. Mech.,* 35(3), 225–232.
7. Guz, A. N. (2002). Critical phenomena in cracking of the interface between two prestressed materials. 1. Problem formulation and basic relations. *Int. Appl. Mech.,* 38(4), 423–431.
8. Guz, A. N. (2002). Critical phenomena in cracking of the interface between two prestressed materials. 2. Exact solution. The case of unequal roots. *Int. Appl. Mech.,* 38(5), 548–555.
9. Guz, A. N. (2002). Critical phenomena in cracking of the interface between two prestressed materials. 3. Exact solution. The case of equal roots. *Int. Appl. Mech.,* 38(6), 693–700.
10. Guz, A. N. (2002). Critical phenomena in cracking of the interface between two prestressed materials. 4. Exact solution. The case of unequal and equal roots. *Int. Appl. Mech.,* 38(7), 806–814.

11. Babich, S. Yu., Glukhov, Yu. P. & Guz, A. N. (2008). Dynamics of a Prestressed Incompressible Layered Half-Space under Moving Load. *Int. Appl. Mech.,* 44(3), 268–285.
12. Babich, S. Yu., Glukhov, Yu. P. & Guz, A. N. (2008). Dynamic Problem for a Prestressed Compressible Layered Half-Space. *Int. Appl. Mech.,* 44(4), 388–405.
13. Babich, S.Yu., Glukhov, Yu.P. and Guz, A.N. (2008). Using Complex Potentials to Determine the Reaction of a Prestressed Two-Layer Elastic Half-Space to a Moving Load. *Int. Appl. Mech.,* 44(5), 481–492.
14. Guz, A. N. (2004). Desing models in linearized solid mechanics. *Int. Appl. Mech.,* 40(1), 1–37.
15. Guz, A. N. (2002). Elastic waves in bodies with initial (residual) stresses. *Int. Appl. Mech.,* 38(1), 23–59.
16. Guz, A. N. (2001). Constructing the three-dimensional theory of stability of deformable bodies. *Int. Appl. Mech.,* 37(1), 1–37.
17. Guz, A. N., & Guz, I. A. (2004). Mixed plane problems in linearized solid mechanics. Exact solutions. *Int. Appl. Mech.,* 40(1), 1–29.
18. Guz, A. N., Menshykov, O. V., Zozulya, V. V. & Guz, I. A. (2007). Contact Problem for the Flat Elliptical Crack under Normally Incident Shear Wave. *CMES.* 17(3), 205–214.
19. Guz, A. N., Rushchitsky, J. J. & Guz, I. A. (2008). Comparative Computer Modeling of Carbon-Polymer Composites with Carbon or Graphite Microfibers or Carbon Nanotubes. *CMES.* 26(3), P. 139–156.
20. Guz, A. N., & Zozulya, V. V. (2007). Investigation of the Effect of Frictional Contact in III Mode Crack under of the SH-Wave Harmonic Load. *CMES.* 22(2), 119–128.
21. Wright, J. P., & Baron, M. L. (1970). Exponentially decaying pressure pulse moving with contact velocity on the surface of a layered elastic material (superseismic half space). *Trans. ASME. Ser. E. J. Appl. Mech.,* 37(1), 148–159.
22. Achenbach, J. D., Keshawa, S. P., & Herrmann, G. (1967). Moving load on a plate resting on an elastic half space. *Trans. ASME. Ser. E. J. Appl. Mech.,* 34(4), 183–189.

Бабич С. Ю., Глухов Ю. П., Лазар В. Ф., Жигуц Ю. Ю. Динаміка двошарового напівпростору з початковим напругами при впливі рухомого навантаження.

У даній статті в рамках лінеаризованої теорії пружності для тіл з початковими напругами розглянуто плоске завдання про обурення, що рухається з постійною швидкістю поверхневим навантаженням двошарового напівпростору з початковими напругами з довільною формою пружного потенціалу. Розв'язок отримано у загальному вигляді для стисливого та стисливого напівпростору та різних умов контакту. Чисельні результати наведені для напівпросторів з матеріалів, що стискається і стискається відповідно з пружним потенціалом гармонійного типу і пружним потенціалом типу Бартенева-Хазановича при жорсткому і ковзному умовах контакту.

Ключові слова: шаруватий напівпростір, початкова напруга, рухоме навантаження.

Список використаної літератури

1. Guz, A. N. Elastic waves in bodies with initial (residual) stresses. Kiev: Publisher "A.S.K", 2004. 672 p.
2. Guz, A. N., Babich S. Yu., Glukhov Yu. P. Statics and dynamics of the elastic grounds with initial (residual) stresses. Kremenchug: Publisher "Kremenchug", 2007. 795 p.
3. Guz, A. N. Dynamic problems of the mechanics of the brittle fracture of materials with initial stresses for moving cracks. 1. Problem statement and general relationships. *Int. Appl. Mech.* 1998. Vol. 34, No 12. P. 1175–1186.
4. Guz, A. N. Dynamic problems of the mechanics of the brittle fracture of materials with initial stresses for moving cracks. 2. Craks of normal separation (Mode I). *Int. Appl. Mech.* 1999. Vol. 35, No 1. P. 1–12.
5. Guz, A. N. Dynamic problems of the mechanics of the brittle fracture of materials with initial stresses for moving cracks. 3. Transverse-Shear (Mode II) and Longitudinal-Shear (Mode III). *Int. Appl. Mech.* 1999. Vol. 35, No 2. P. 109–119.

6. Guz, A. N. Dynamic problems of the mechanics of the brittle fracture of materials with initial stresses for moving cracks. 4. Wedge problems. *Int. Appl. Mech.* 1999. Vol. 35, No 3. P. 225–232.
7. Guz, A. N. Critical phenomena in cracking of the interface between two prestressed materials. 1. Problem formulation and basic relations. *Int. Appl. Mech.* 2002. Vol. 38, No 4. P. 423–431.
8. Guz, A. N. Critical phenomena in cracking of the interface between two prestressed materials. 2. Exact solution. The case of unequal roots. *Int. Appl. Mech.* 2002. Vol. 38, No 5. P. 548–555.
9. Guz, A. N. Critical phenomena in cracking of the interface between two prestressed materials. 3. Exact solution. The case of equal roots. *Int. Appl. Mech.* 2002. Vol. 38, No 6. P. 693–700.
10. Guz, A. N. Critical phenomena in cracking of the interface between two prestressed materials. 4. Exact solution. The case of unequal and equal roots. *Int. Appl. Mech.* 2002. Vol. 38, No 7. P. 806–814.
11. Babich S. Yu., Glukhov Yu. P., Guz A. N. Dynamics of a Prestressed Incompressible Layered Half-Space under Moving Load. *Int. Appl. Mech.* 2008. Vol. 44, No 3. P. 268–285.
12. Babich S. Yu., Glukhov Yu. P., Guz A. N. Dynamic Problem for a Prestressed Compressible Layered Half-Space. *Int. Appl. Mech.* 2008. Vol. 44, No 4. P. 388–405.
13. Babich S. Yu., Glukhov Yu. P., Guz A. N. Using Complex Potentials to Determine the Reaction of a Prestressed Two-Layer Elastic Half-Space to a Moving Load. *Int. Appl. Mech.* 2008. Vol. 44, No 5. P. 481–492.
14. Guz, A. N. Desing models in linearized solid mechanics. *Int. Appl. Mech.* 2004. Vol. 40, No 1. P. 1–37.
15. Guz, A. N. Elastic waves in bodies with initial (residual) stresses. *Int. Appl. Mech.* 2002. Vol. 38, No 1. P. 23–59.
16. Guz, A. N. Constructing the three-dimensional theory of stability of deformable bodies. *Int. Appl. Mech.* 2001. Vol. 37, No 1. P. 1–37.
17. Guz, A. N., Guz, I. A. Mixed plane problems in linearized solid mechanics. Exact solutions. *Int. Appl. Mech.* 2004. Vol. 40, No 1. P. 1–29.
18. Guz, A. N., Menshykov O. V., Zozulya V. V., Guz I. A. Contact Problem for the Flat Elliptical Crack under Normally Incident Shear Wave. CMES. 2007. Vol. 17, No 3. P. 205–214.
19. Guz, A. N., Rushchitsky J. J., Guz I. A. Comparative Computer Modeling of Carbon-Polymer Composites with Carbon or Graphite Microfibers or Carbon Nanotubes. CMES. 2008. Vol. 26, No 3. P. 139–156.
20. Guz, A. N., Zozulya V.V. Investigation of the Effect of Frictional Contact in III Mode Crack under of the SH-Wave Harmonic Load. CMES. 2007. Vol. 22, No 2. P. 119–128.
21. Wright J. P., Baron M. L. Exponentially decaying pressure pulse moving with contact velocity on the surface of a layered elastic material (superseismic half space). *Trans. ASME.* Ser. E. *J. Appl. Mech.* 1970. Vol. 37, No 1. P. 148–159.
22. Achenbach J. D., Keshawa S. P., Herrmann, G. Moving load on a plate resting on an elastic half space. *Trans. ASME.* Ser. E. *J. Appl. Mech.* 1967. Vol. 34, No 4. P. 183–189.

Одержано 08.04.2022