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ON A NUMBER OF EMIGRANTS IN DECOMPOSABLE AGE-DEPENDENT BRANCHING PROCESSES

Decomposable branching process can be viewed as stohastic model for the population with N types of individuals, splitted into several groups $G_1, G_2, \ldots, G_n$, $n \leq N$, where each group occupies its own island. Individual of group G_i may immidiately after birth emigrate to island, occupied by group with higher index or stay on the same island. In given paper we consider case with two groups G_1 and G_2. Each individual has random life duration and distribution of its progeny depends on its age.

We establish asymptotic behaviour of processes that count number of emigrated individuals, depending on criticality of branching subprocess, generated by group G_1.

Keywords: branching process, stochastic additive functional, critical branching process, Perron root, moments.

1. Introduction. We will provide short description of decomposable multitype age-dependent branching process with variable transition probabilities. Review of multi-type age-dependent processes with variable transition probabilities can be found in [1], chapter 8, while decomposable branching processes were studied, for example, in [2], [3] and [4]. Description of probability space could be given analogically to [5], chapter 6.

Consider a population consisting of n types $T_1, T_2, \ldots, T_n$ Each $T_i - th$ type particle has random life duration τ_i with distribution function

$$P(\tau_i \leq t) = G^i(t), G^i(0+) = 0$$

We will assume that $G^i(t)$ are absolutely continuous.

Types of particles can be divided into two groups: C_1 which includes particles of types $T_1, T_2, \ldots, T_r$ and C_2 which includes particles (individuals) of types $T_{r+1}, T_{r+2}, \ldots, T_n$, $1 \leq r < n$. Particles from C_2 at the end of their lives transorm into any number of particles from their own group, while particles from C_1 transform into any number of particles of any type. Direct C_2 types descendants of particles from C_1 group types we will call 'emigrants'. Establishing asymtotic behaviour of processes, which describe number of emigrants is the goal of this paper.

Conditional probability (if transformation took place when the age attained by the original particle was u) $p^i_\alpha(u)$ of transformation into a set consisting of α_i $T_i - th$ type particles, $i = \overline{1, n}$, where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ is n - dimensional vector where the components are non-negative integers. Evolution of particle is defined by joint distribution of random variable τ_i and random vector $v_i = (v_i^1, \ldots, v_i^n)$, which characterise progeny of this particle.

$$P(\tau_i \in B, v_i = \alpha) = \int_B p^i_\alpha(u) dG^i(u).$$

Vector $\mu_i(t) = (\mu_i^1(t), \dots, \mu_i^n(t))$ denotes number of particles of types $T_1, T_2, \dots, T_n$ at the moment t, under the condition that at initial moment there existed one $T_i - th$ type particle. And let's also assume, that vectors $\mu^i(t)$ are right continuous.

Let's denote by $P^i(*)$ and $E^i(*)$ denote conditional probability and conditional expectaion respectively, under condition that at initial moment of time there existed one particle of type T_i.

Let's also introduce generating functions

$$h^i(t, s) = \sum_\alpha p^i_\alpha(t) s^\alpha$$

and

$$F^i(t, s) = \sum_\alpha P^i(\mu_i(t) = \alpha) s^\alpha, i = \overline{1, n},$$

$$s = (s_1, \dots, s_n), s^\alpha = s_1^{\alpha_1} \cdots s_n^{\alpha_n},$$

$$F(t, s) = (F^1(t, s), \dots, F^n(t, s)), h(t, s) = (h^1(t, s), \dots, h^n(t, s)).$$

Generating functions $F^i(t, s)$ satisfy with $s \leq 1$ (componentwise) and $t \geq 0$ next system of integral equations [1]

$$F^i(t, s) = s_i(1 - G^i(t)) + \int_0^t h^i(u, F(t - u, s)) dG^i(u), i = \overline{1, n}. \tag{1}$$

2. Preliminaries. Define

$$a^i_j(u) = \frac{\partial h^i(u, s)}{\partial s_j}|_{s=1}, A^i_j = \int_0^\infty a^i_j(u) dG^i(u),$$

$$b^i_{jk}(u) = \frac{\partial h^i(u, s)}{\partial s_j \partial s_k}|_{s=1}, B^i_{jk} = \int_0^\infty b^i_{jk}(u) dG^i(u).$$

We will call described above branching process (b.p.) ξ. We assume that matrix $A = \left\|A^i_j\right\|_{i,j=\overline{1,n}}$ of moments has a form $\begin{bmatrix} A_1 & A_{12} \\ \mathbf{0} & A_2 \end{bmatrix}$, where matrices $A_1 = \left\|A^i_j\right\|_{i,j=\overline{1,r}}$ and $A_2 = \left\|A^i_j\right\|_{i,j=\overline{r+1,n}}$ are irreducable, $A_{12} = \left\|A^i_j\right\|_{i=\overline{1,r}, j=\overline{r+1,n}}$, $\mathbf{0}$=(n-r)*r zero matrix.

Since our goal is to describe number of emigrants, we will mainly focus on matrix A_1.

Let

$$A^l_{\rho k} = \int_0^\infty u e^{-\rho u} a^l_k(u) dG^l(u), B^l_{\rho jk} = \int_0^\infty e^{-2\rho u} b^l_{jk}(u) dG^l(u),$$

$$M^k = \int_0^\infty u dG^k(u), M^{lk}_a = \int_0^\infty u a^l_k(u) dG^l(u),$$

$$B = \sum_{l,k,m=1}^r B^l_{mk} v^l u^k u^m, M_a = \sum_{l,k=1}^r M^{lk}_a v^l u^k, M_j = \sum_{l=1}^r v^l \int_0^\infty a^l_j(u) dG^l(u),$$

where ρ denotes such number, that perron root of matrix $\left\|\int_0^\infty e^{-\rho u} a^l_k(u) dG^l(u)\right\|_{i,j=\overline{1,r}}$ equals to one, $u_\rho = (u^1_\rho, u^2_\rho, \dots, u^r_\rho)$ and $v_\rho = (v^1_\rho, v^2_\rho, \dots, v^r_\rho)$ denote right and left eigenvectors of this matrix, $(v_\rho, u_\rho) = (v_\rho, \mathbf{1}) = \sum_{k=1}^{r} v_\rho^k = 1$ (in critical case $\rho = 0$ and we denote $u_\rho = u, v_\rho = v$).

To simplify further investigation we will introduce process ξ', which consists only of particles from group C_1. By $\nu_i(t) = (\nu_i^{r+1}(t), \dots, \nu_i^n(t))$ we will denote number of particles of types from group C_2 (emigrants), produced by C_1 type particles by t. Processes $\nu_i(t), i = \overline{1,r}$ can be seen as 'product' of C_1 type particles or multdimensional additive functional from branching process ξ' (see [7]).

Consider vector $\tilde{\mu}_i(t) = (\mu_i^1(t), \dots, \mu_i^r(t), \nu_i^{r+1}(t), \dots, \nu_i^n(t)), i = \overline{1,n}$.

By $\tilde{F}^i(t,s)$ we will denote generating functions of this vectors. It is easy to see that $F^j(t,s) = s_j$ for $j = \overline{r+1,n}$ and formula (1) is valid for $\tilde{F}^i(t,s), i = \overline{1,r}$ with $\tilde{F}(t,s) = (\tilde{F}^1(t,s), \dots, \tilde{F}^n(t,s))$.

According to [1], we have next formulas for moments

$$A_j^i(t) = \frac{\partial F^i(t,s)}{\partial s_j} = \frac{\partial \tilde{F}^i(t,s)}{\partial s_j},$$

$$B_{jk}^i(t) = \frac{\partial^2 F^i(t,s)}{\partial s_j \partial s_k} = \frac{\partial^2 \tilde{F}^i(t,s)}{\partial s_j \partial s_k},$$

$$A_j^i(t) = \delta_j^i(1 - G^i(t) + \sum_{k=1}^{r} \int_0^t A_j^k(t-u) a_k^i(u) dG^i(u),$$

$$B_{jk}^i(t) = \sum_{l=1}^{r} \int_0^t B_{jk}^l(t-u) a_l^i(u) dG^i(u) + \sum_{m,l=1}^{r} \int_0^t A_j^l(t-u) A_k^m(t-u) b_{lm}^i(u) dG^i(u),$$

for $i, j, k = \overline{1,r}$.

For $A_l^i(t), B_{jl}^i(t)$ and $B_{ll}^i(t)$, were $i, j = \overline{1,r}, l = \overline{r+1,n}$ we will have

$$A_l^i(t) = \int_0^t a_l^i(u) dG^i(u) + \sum_{k=1}^{r} \int_0^t A_l^k(t-u) a_k^i(u) dG^i(u), \tag{2}$$

$$B_{jl}^i(t) = \sum_{m=1}^{r} \int_0^t B_{jl}^m(t-u) a_m^i(u) dG^i(u) +$$

$$+ \sum_{m,d=1}^{r} \int_0^t A_j^m(t-u) A_l^q(t-u) b_{mq}^i(u) dG^i(u), \tag{3}$$

$$B_{ll}^i(t) = \sum_{m=1}^{r} \int_0^t B_{ll}^l(t-u) a_m^i(u) dG^i(u) +$$

$$+ \sum_{m,q=1}^{r} \int_0^t A_l^m(t-u) A_l^q(t-u) b_{mq}^i(u) dG^i(u). \tag{4}$$

Next theorem is crucial for establishing asymtotic behaviour of moments. Proof of this theorem can be found in [6].

Theorem 1. *Let* $M(dy) = \left\|m_{ij}(dy)\right\|_{i,j=\overline{1,r}}$ *be* r**r square matrix, components of which are finite non-negative measures on* $[0,+\infty)$ *and let vector function* $g(x) = (g_1(x),\ldots,g_r(x))$ *be such that for some* $\gamma \geq 0$ *holds* $\sup\limits_{x\in\mathbb{R}} \frac{g(x)}{max\{1,x^\gamma\}} < \infty$ *and* $\frac{g(x)}{x^\gamma} \xrightarrow{x\to+\infty} c = (c_1,\ldots,c_r)$. *If Perron root of* $M[0,+\infty)$ *equals to 1 and* $\int\limits_0^\infty u m_{ij}(du) < \infty$, *then*

$$\frac{1}{x^{1+\gamma}}\int_0^x g(x-y)dH(y) \xrightarrow{x\to+\infty} \frac{c}{(1+\gamma)a}\left\|u^i v^j\right\|_{i,j=\overline{1,r}},$$

where $H(y)$ *is renewal matrix, which corresponds to matrix* $M(dy)$, u *and* v *are right and left eigenvectors of* $M[0,+\infty)$, $a = \left(v, \int\limits_0^\infty yM(dy)u\right)$.

Applying **Theorem1** to (2)–(4), one can show that, in critical case, if M^j, M_a^{jk}, B_{lk}^j are finite, then

$$A_l^i(t) \sim \frac{u^i M_l t}{M_a}, B_{jl}^i(t) \sim B\frac{u^i v^j M^j M_l t^2}{2(M_a)^2}, B_{ll}^i(t) \sim B\frac{u^i (M_l)^2 t^3}{3(M_a)^3}. \tag{5}$$

We will also need next lemma to prove **Theorem 2**.

Lemma 1. *If for random variables* $X(t)$ *and* $Y(t)$ *following conditions are satisfied:*

a) $X(t) \xrightarrow{t\to+\infty} X$ *in distribution;*

b) $\lim\limits_{t\to\infty}(X(t)-Y(t))^2$,

then $Y(t) \xrightarrow{t\to+\infty} X$ *in distribution.*

Proof. Indeed, from condition b) we get that $Y(t) = X(t) + \theta(t)$, where $\theta(t)$ tends to zero in square mean, therefore in distribution, so

$$\lim_{t\to\infty} E\left(exp\left\{\ i\beta Y(t)\ \right\}\right) = \lim_{t\to\infty} E\left(exp\left\{\ i\beta(X(t)+\theta(t))\ \right\}\right) = E\left(exp\left\{\ i\beta X\ \right\}\right).$$

3. Main results. Analogy of theorem 2 (and corollary 1) [7] takes place, which also can be proven using slight modification of results obtained in [2]:

Claim 1. *If process* ξ *is supercritical,* $A_{\rho k}^l(u)$ *and* $B_{\rho jk}^l$ *are finite,* $i,j,k = \overline{1,n}$, *than random variables (r.v.)* $\frac{e^{-\rho t}\mu_1^i(t)}{K_1},\ldots,\frac{e^{-\rho t}\mu_r^i(t)}{K_r},\frac{e^{-\rho t}\nu_{r+1}^i(t)}{K_{r+1}},\ldots,\frac{e^{-\rho t}\nu_n^i(t)}{K_n}$ *converge as* $t\to\infty$ *in square mean to the same limit* μ^i, *where*

$$K_j = \frac{v_\rho^j \int_0^\infty e^{-\rho t}(1-G^i(t))dt}{\rho\sum\limits_{k,m=1}^r v_\rho^k u_\rho^m \int_0^\infty te^{-\rho t}a_m^i(t)dG^k(t)}$$

$$K_l = \frac{\sum\limits_{k=1}^r v_\rho^k \int_0^\infty e^{-\rho t}\int_0^t a_l^i(u)dG^i(u)dt}{\sum\limits_{k,m=1}^r v_\rho^k u_\rho^m \int_0^\infty te^{-\rho t}a_m^i(t)dG^k(t)},\ j=\overline{1,r},\ l=\overline{r+1,n}$$

Furthermore, Laplace transform $\phi^l(s)$ *of limit r.v.* $\tilde{\mu}^l$ *satisfies equation*

$$\phi^l(s) = \int_0^\infty h^l\left(u, \phi(se^{-\rho u})\right) dG^l(u),$$

with initial condition $\phi(0) = 1, \phi'(0) = -1,$ *where* $\phi(s) = (\phi^1(s), \ldots, \phi^n(s)).$

Corollary 1. *If conditions of theorem 1 are satisfied, than distributions*

$$P^i\left(\frac{\sum\limits_{k=r+1}^{n} \nu_l^i(t)}{\sum\limits_{k=1}^{r} \mu_k^i(t)} \le x | \sum_{k=1}^{r} \mu_k^i(t) > 0\right), \ i = \overline{1, r}$$

converge to the degenerate distribution, localized at the point

$$\frac{\sum\limits_{l=r+1}^{n} \sum\limits_{k=1}^{r} v_\rho^k \int_0^\infty e^{-\rho t} \int_0^t a_l^i(u) dG^i(u) dt}{\rho \sum\limits_{l=1}^{r} v_\rho^l \int_0^\infty e^{-\rho t}(1 - G^i(t)) dt}.$$

In order to prove next theorem we will compare processes $\nu_l^i(t)$ with processes $N^i(t)$ — total number of particles born by the moment of time t, if at the moment $t = 0$ there existed one particle of type T_i.

Let $N_j^i(t)$ denote number of particles of type T_j, born by t, then $N^i(t) = \sum\limits_{j=1}^{r} N_j^i(t)$. It is known [8], that

$$E\left(exp\left\{\ i \sum_{j=1}^{r} \beta^j N_j^i(t)/v^j t^2\ \right\} | \sum_{j=1}^{r} \mu_j^i(t)\right) \xrightarrow{t\to+\infty}$$

$$\xrightarrow{t\to+\infty} \left(2B \sum_{j=1}^{r} \beta^j\right)^{1/2} / M_a \left(sh\left(\left(2B \sum_{j=1}^{r} \beta^j\right)^{1/2} / M_a\right)\right).$$

Then by letting $\beta^j = v^j \beta$, and since $\sum\limits_{j=1}^{r} v^j = 1$, we get

$$E\left(exp\left\{\ i\beta N^i(t)/t^2\ \right\} | \sum_{j=1}^{r} \mu_j^i(t)\right) \xrightarrow{t\to+\infty}$$

$$\xrightarrow{t\to+\infty} (2B\beta)^{1/2} / M_a \left(sh\left((2B\beta)^{1/2} / M_a\right)\right). \qquad (6)$$

Also from [9], pp. 464–465, we can establish asymptotic behaviour of moments $E^i(N_i(t))$ and $E^i(N_i^2(t))$:

$$E^i(N_i(t)) \sim \frac{u^i t}{M_a}, E^i(N_i^2(t)) \sim B\frac{u^i t^3}{3(M_a)^3}.$$

Theorem 2. *If the following conditions are satisfied:*

i) integrals M^j, M_a^{jk}, B_{jk}^i *are finite;*

ii) $\int_0^\infty \int_t^\infty a_l^m(u)dG^m(u)dt = k_l^m < +\infty;$
iii) $\lim_{t\to\infty} t^2 \int_t^\infty a_m^k(u)dG^m(u) < +\infty, \lim_{t\to\infty} t^2 \left(1 - G^l(t)\right) < +\infty,$
$m, k, j = \overline{1, r}, l = \overline{r+1, n},$ *then*

$$E^i \left(exp\left\{ \ i\beta\mu_l^i(t)/t^2 \ \right\} | \sum_{j=1}^r \mu_j^i(t) > 0 \right) \xrightarrow{t\to+\infty}$$

$$\xrightarrow{t\to+\infty} (2BM_0\beta)^{1/2}/M_a \left(sh\left((2BM_0\beta)^{1/2}/M_a \right) \right).$$

Proof. As has been mentioned in (5), moments $A_l^j(t) \sim \frac{u^i M_l t}{M_a}$, $B_{ll}^i(t) \sim \sim B\frac{u^i(M_l)^2 t^3}{3(M_a)^3}$, $l = \overline{1, r}$.

Now we will obtain asymptotic behavior of moments $E^i\left(N^i(t)\nu_l^i(t)\right)$.

Let

$$F^i(t, z, s) = E^i\left(e^{zN^i(t)}s^{\tilde{\mu}^i(t)}\right), z \le 0, i = \overline{1, r},$$

$$F(t, z, s) = (F^1(t, z, s), \ldots, F^r(t, z, s)).$$

Let also

$$D^i(t) = \frac{\partial F^i(t, z, s)}{\partial z}|_{s=1,z=0}, D_l^i(t) = \frac{\partial^2 F^i(t, z, s)}{\partial z\partial s_l}|_{s=1,z=0},\ l = \overline{r+1, n}.$$

Similarly to [5], we can derive formula

$$F^i(t, z, s) = e^z\left(s_i\left(1 - G^i(t)\right) + \int_0^t h^i\left(u, F(t-u, z, s)\right)dG^i(u)\right). \qquad (7)$$

Differentiating both sides of (7) first with respect to z at point 0, than with respect to s_0 at the point $s = \underbrace{(1, \ldots, 1)}_{n}$ we get

$$D_l^i(t) = \int_0^t a_l^i(u)dG^i(u) + \sum_{k=1}^r \int_0^t A_l^k(t-u)a_k^i(u)dG^i(u) +$$

$$+ \sum_{j,k=1}^r \int_0^t A_l^k(t-u)D^i(t-u)b_{jk}^i(u)dG^i(u) + \sum_{k=1}^r \int_0^t D_l^i(t-u)a_k^i(u)dG^i(u).$$

Using the results we obtained above, we see that

$$\int_0^t a_l^i(u)dG^i(u) + \sum_{k=1}^r \int_0^t A_l^k(t-u)a_k^i(u)dG^i(u) = o(t^2),$$

$$\sum_{j,k=1}^r \int_0^t A_l^k(t-u)D^i(t-u)b_{jk}^i(u)dG^i(u) \sim \sum_{j,k=1}^r B_{jk}^i u^j u^k \frac{M_l t^2}{(M_a)^2}.$$

So again, using **Theorem1**, we obtain that $D_l^i(t) \sim B\frac{u^i M_l t^3}{3(M_a)^3}$.

Conditions ii) and iii) allows us to claim ([10]), that $P^i\left(\sum_{k=1}^{r}\mu_k^i(t)>0\right)\sim\frac{2M_a}{Bt}$ and so

$$E\left(\left(\frac{(M_a)^2N^i(t)}{t^2}-\frac{(M_a)^2\nu_l^i(t)}{M_lt^2}\right)^2 | \sum_{k=1}^{r}\mu_k^i(t)>0\right)\sim$$

$$\sim\frac{BM_au^i/3t-2BM_au^i/3t+BM_au^i/3t}{2M_a/Bt}=0.$$

From here, (6) and from **Lemma1** we get the result.

Corollary 2. *If conditions of theorem 1 are satisfied, than distributions*

$$P^i\left(\frac{\nu_l^i(t)}{N^i(t)}\leq x | \sum_{k=1}^{r}\mu_k^i(t)>0\right),$$

converge to the degenerate distribution, localized at the point $\frac{1}{M_l}$, *where,* $i=\overline{1,r}$, $l=$ $=\overline{r+1,n}$.

Analogy of theorem 1, [7] also takes place.

Claim 2. *Process* ξ *is critical or subcritical if and only if random variables* $\mu_l^j(t)$ *converge with probability 1 as* $t\to+\infty$ *to finitely valued r. v.* $\nu_l^j, j=\overline{1,r}, l=$ $=\overline{r+1,n}$.

Furthermore, vector characteristic function of limit r.v. $\nu_l=(\nu_l^1,\dots,\nu_l^r)$ —

$$E\left(e^{i\beta\nu_l}\right)=\left(E\left(e^{i\beta\nu_l^1}\right),\dots,E\left(e^{i\beta\nu_l^r}\right)\right),$$

satisfies integral equation:

$$E\left(e^{i\beta\nu_l}\right)=\left(\int_0^{+\infty}h^1\left(u,\tilde{E}\left(e^{i\beta\nu_l}\right)\right)dG^1(u),\dots,\int_0^{+\infty}h^r\left(u,\tilde{E}\left(e^{i\beta\nu_l}\right)\right)dG^n(u)\right),$$

where

$$\tilde{E}\left(e^{i\beta\nu_l}\right)=\left(E\left(e^{i\beta\nu_l^1}\right),\dots,E\left(e^{i\beta\nu_l^r}\right),s_{r+1},\dots,s_n\right).$$

Proof. Proof is analogical to theorem 1 in [7]. It is only worth to note that if process is supercritical, then there exist $i=\overline{1,r}$, such that

$$0<\lim_{t\to+\infty}P^i\left(\sum_{k=1}^{r}\mu_k^i(t)>0\right)=\lim_{t\to+\infty}Q^i(t)=q^i<1,$$

and process $N^i(t)\xrightarrow{t\to+\infty}+\infty$ (see chapter 6, [5]) with probability q^i, so the strong law of large numbers holds for $\frac{v_l^i}{N^t}$.

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Лисецький Т. Б. Про кількість емігрантів в розкладному гіллястому процесі з перетвореннями, залежними від віку.

Розкладний гіллястий процес можна розглядати як стохастичну модель популяції з N типами індивідиумів, розділених на декілька підгруп $G_1, G_2, \ldots, G_n$, $n \leq N$, де кожна група населяє окремий острів. Індивідиум з групи G_i може одразу після народження емігрувати на острів, населений групою з вищим індексом, або залишитись на своєму острові. В даній статті розглядається випадок з двома групами G_1 та G_2. Кожна особа має випадкову тривалість життя, а розподіл її потомства залежить від її віку.

Ми досліджуємо асимптотичну поведінку процесів, які рахують кількість частинок, що емігрували, в залежності від критичності гіллястого підпроцесу, породженого групою G_1.

Ключові слова: гіллястий процес, стохастичний адитивний функціонал, критичний гіллястий процес, перонів корінь, моменти.

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