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QUADRATIC-ACCURACY ONE-BULLET SILENT DUEL ON THE UNIFORM LATTICE: PURE STRATEGY SOLUTIONS EXIST WHEN THE DUELIST HAS AT MOST EIGHT TIME MOMENTS

The one-bullet silent duel defined on the uniform lattice is considered, in which each of the two duelists shoots with quadratic accuracy. The duel is a symmetric matrix game whose optimal value is 0, and each of the duelists has the same optimal behavior, whether it is in pure or mixed strategies. The task is to determine optimal time moments for the duelist to shoot depending on the number of the duel time moments. It is proved that an optimal time moment in the duel exists only if the duelist has three to eight time moments to shoot, except for seven time moments, and the optimal time moment is single. The quadratic-accuracy duelist's optimal time moment is the duel end moment in the 3×3 and 4×4 duels, is the penultimate time moment in the 5×5 and 6×6 duels, and is the time moment preceding the penultimate time moment in the 8×8 duel. Compared to the linear-accuracy duel, which does not have an optimal time moment for six time moments and for no fewer than eight time moments, the quadratic-accuracy duel provides the duelist with a single one-step ("instant") solution for six and eight time moments instead of seven ones. Another difference is that the quadratic-accuracy duelist's optimal time moment gravitates more towards the duel end moment, unlike it is the duel span middle in the linear-accuracy duel.

Keywords: one-bullet silent duel, uniform lattice, quadratic accuracy, matrix game, optimal time moment.

1. One-bullet silent duels.

One-bullet silent duels are a large part of timing games that model competitive interactions between two equal participants (competitors, players, or duelists), where the general purpose is to take one timely action (to shoot the single bullet) [1]. The timeliness means shooting earlier than the other duelist, but shooting later grants a better benefit [2]. The duel time span is usually divided into equidistant intervals whose endpoints are the time moments at which the duelist is allowed to (legitimately) shoot [3]. Thus, if there are N successive time moments of possible shooting, then they are represented as a set [4]

$$T_N = \{t_q\}_{q=1}^{N} = \left\{ \frac{q-1}{N-1} \right\}_{q=1}^{N} \subset [0;\ 1] \quad \text{for} \quad N \in \mathbb{N} \backslash \{1,\ 2\} . \tag{1}$$

With (1), the respective one-bullet silent duel is a finite zero-sum game [5,6]

$$\langle X_N,\ Y_N,\ \mathbf{U}_N \rangle = \left\langle \{x_i\}_{i=1}^{N},\ \{y_j\}_{j=1}^{N},\ \mathbf{U}_N \right\rangle \tag{2}$$

of timing with the duelists' pure strategy sets

$$X_N = \{x_i\}_{i=1}^{N} = \left\{ \frac{i-1}{N-1} \right\}_{i=1}^{N} = T_N \subset [0;\ 1] \tag{3}$$

and

$$Y_N = \{y_j\}_{j=1}^{N} = \left\{ \frac{j-1}{N-1} \right\}_{j=1}^{N} = T_N \subset [0;\ 1] \tag{4}$$

by a skew-symmetric payoff matrix

$$\mathbf{U}_N = [u_{ij}]_{N \times N} = [-u_{ji}]_{N \times N} = -\mathbf{U}_N^{\mathrm{T}}. \tag{5}$$

Hence, duel (2) is defined on uniform lattice

$$X_N \times Y_N = \{x_i\}_{i=1}^{N} \times \{y_j\}_{j=1}^{N} = \\ = \left\{ \frac{i-1}{N-1} \right\}_{i=1}^{N} \times \left\{ \frac{j-1}{N-1} \right\}_{j=1}^{N} \subset [0;\ 1] \times [0;\ 1]. \tag{6}$$

The skew-symmetry of payoff matrix (5) reflects the equality of the duelists, by which the optimal game value is 0, and each of the duelists has the same optimal behavior (whether it is in pure or mixed strategies) [2,7]. Inasmuch as one-bullet silent duels model hardly repeatable environments, the common task is to determine optimal time moments (pure strategies) for the duelist [6,8,9]. If there are no optimal time moments at the duelist, i. e. duel (2) is not solved in pure strategies (but, certainly, is solved in mixed strategies), the respective duel configuration (like the number of time moments of possible shooting, the accuracy function of the duelist) must be rectified to obtain an optimal time moment [10]. This is practically always done to comply with non-repeatability of the modeled environment [5,11,12].

2. Optimal time moments in linear-accuracy duel. Article [13] determined that linear-accuracy one-bullet silent duel (2) on uniform lattice (6), where

$$u_{ij} = x_i - y_j + x_i y_j \operatorname{sign}(y_j - x_i) \quad \text{for} \quad i = \overline{1,\ N} \quad \text{and} \quad j = \overline{1,\ N}, \tag{7}$$

is solved in pure strategies only when $N \in \{3,\ 4,\ 5,\ 7\}$. In the 3×3 linear-accuracy duel, where

$$T_3 = \{t_1,\ t_2,\ t_3\} = \left\{ 0,\ \frac{1}{2},\ 1 \right\}, \tag{8}$$

the duelist has two optimal moments, which are $t_2 = \frac{1}{2}$ and $t_3 = 1$. In the 4×4 linear-accuracy duel, where

$$T_4 = \{t_1,\ t_2,\ t_3,\ t_4\} = \left\{ 0,\ \frac{1}{3},\ \frac{2}{3},\ 1 \right\}, \tag{9}$$

only third moment $t_3 = \frac{2}{3}$ is optimal. In the 5×5 linear-accuracy duel, where

$$T_5 = \{t_1,\ t_2,\ t_3,\ t_4,\ t_5\} = \left\{ 0,\ \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1 \right\}, \tag{10}$$

only third moment $t_3 = \frac{1}{2}$ is optimal as well. The biggest linear-accuracy duel having a pure strategy solution is the 7×7 one, where

$$T_7 = \{t_1,\ t_2,\ t_3,\ t_4,\ t_5,\ t_6,\ t_7\} = \left\{ 0,\ \frac{1}{6},\ \frac{1}{3},\ \frac{1}{2},\ \frac{2}{3},\ \frac{5}{6},\ 1 \right\} \tag{11}$$

and only fourth moment $t_4 = \frac{1}{2}$ is optimal. Therefore, the duelist in duel (2) on uniform lattice (6) by (3) — (5), (7), if it is solved in pure strategies, mostly has to shoot at the middle of the duel span. The only exception is the 4×4 duel, which does not have the middle of the duel span in set (9). The generalization of the linear-accuracy duel on uniform lattice (6) was studied in [14], where manipulations with the linear-accuracy proportionality factor allow to have an optimal time moment (although such manipulations are not always realizable in practice). Now, the task is to determine optimal time moments for quadratic-accuracy one-bullet silent duel (2) on uniform lattice (6) by (3) — (5), where

$$u_{ij} = x_i^2 - y_j^2 + x_i^2 y_j^2 \operatorname{sign}(y_j - x_i) \quad \text{for } \ i = \overline{1,\ N} \ \text{ and } \ j = \overline{1,\ N}. \tag{12}$$

The quadratic accuracy is more plausible than the linear one. Besides, compared to the linear accuracy, as $t_q^2 < t_q$ for $t_q \in (0;\ 1)$, the quadratic accuracy implies that the duelist is relatively worse at shooting.

3. When an optimal time moment exists. Duel (2) is a symmetric matrix game, in which the existence of a pure strategy solution is equivalent to a pure strategy saddle point in payoff matrix (5). Owing to the skew-symmetry of matrix (5), whose main diagonal is of N zeros and optimal game value is 0, any saddle point of matrix (5) is a zero entry in a nonnegative row and a nonpositive column. Thus, if row i^* of matrix (5) by $i^* \in \left\{\overline{1,\ N}\right\}$ is nonnegative, then row i^* contains a saddle point on the main diagonal [5,6]. A symmetric reasoning is true for columns: if column i^* of matrix (5) by $i^* \in \left\{\overline{1,\ N}\right\}$ is nonpositive, then column i^* contains a saddle point on the main diagonal [1, 2]. This saddle point corresponds to an optimal pure strategy or an optimal time moment t_{i^*} of the duelist in an optimal pure strategy situation

$$\{x_{i^*},\ y_{i^*}\} = \{t_{i^*},\ t_{i^*}\}. \tag{13}$$

If row i^* contains only positive entries, except for the main diagonal entry $u_{i^*i^*} = 0$, all the other $N-1$ rows of column i^* contain negative entries, and thus row i^* contains a single saddle point corresponding to the single optimal pure strategy situation (13) or the single optimal time moment t_{i^*} in this duel. If two time moments t_{i^*} and t_{j_*} are optimal by $i^* \in \left\{\overline{1,\ N}\right\}$, $j_* \in \left\{\overline{1,\ N}\right\}$, and $i^* \neq j_*$, then there are at least four optimal pure strategy situations: situation (13) and situations

$$\{x_{i^*},\ y_{j_*}\} = \{t_{i^*},\ t_{j_*}\},$$

$$\{x_{j_*},\ y_{i^*}\} = \{t_{j_*},\ t_{i^*}\},$$

$$\{x_{j_*},\ y_{j_*}\} = \{t_{j_*},\ t_{j_*}\}.$$

Obviously, time moment t_{i^*} is not optimal if row i^* contains a negative entry or column i^* contains the positive entry. Therefore, it is conventionally possible to conclude on optimal time moment existence by studying nonnegative rows or nonpositive columns of payoff matrix (5). In particular, inasmuch as

$$u_{1j} = -y_j^2 < 0 \ \ \forall\, j = \overline{2,\ N}$$

then the first row of matrix (5) is not an optimal strategy of the first duelist, and thus the starting moment $t_1 = 0$ is never optimal in this duel, whichever the number of time moments is.

4. The most trivial duel.

Theorem 1. *In the most trivial one-bullet silent duel* (2)

$$\langle X_3,\ Y_3,\ \mathbf{U}_3\rangle = \left\langle \left\{0,\ \frac{1}{2},\ 1\right\},\ \left\{0,\ \frac{1}{2},\ 1\right\},\ \mathbf{U}_3 \right\rangle \tag{14}$$

by (3) — (5), (12) *for* $N = 3$ *and* (8), *the duelist has the single optimal time moment* $t_3 = 1$.

Proof. Upon plugging elements of set (8) into (12) for $N = 3$, the respective payoff matrix (5) is

$$\mathbf{U}_3 = [u_{ij}]_{3\times3} = \begin{bmatrix} 0 & -\frac{1}{4} & -1 \\ \frac{1}{4} & 0 & -\frac{1}{2} \\ 1 & \frac{1}{2} & 0 \end{bmatrix}. \tag{15}$$

The third row of matrix (15) is positive, except for the main diagonal entry $u_{33} = 0$, so time moment $t_3 = 1$ is the single optimal one.

5. Optimal time moment in bigger duels. Now, we are about to consider 4×4 and bigger duels. The following assertion includes also the result of Theorem 1.

Theorem 2. *In duel* (2) *by* (3) — (5), (12), *the duelist has an optimal time moment if the number of moments to shoot is between three and eight, except for seven time moments* (11). *The optimal time moment is the single one:* $t_3 = 1$ *is the single optimal time moment in the* 3×3 *duel,* $t_4 = 1$ *is the single optimal time moment in the* 4×4 *duel,* $t_4 = \frac{3}{4}$ *is the single optimal time moment in the* 5×5 *duel,* $t_5 = \frac{4}{5}$ *is the single optimal time moment in the* 6×6 *duel, and* $t_6 = \frac{5}{7}$ *is the single optimal time moment in the* 8×8 *duel.*

Proof. Owing to Theorem 1, time moment $t_3 = 1$ is the single optimal one in the duel with three time moments. For duels with greater number of time moments, suppose that time moment

$$t_n = \frac{n-1}{N-1} \text{ for some } n \in \left\{\overline{2,\ N-1}\right\} \text{ by } N \in \mathbb{N}\backslash\{1,\ 2,\ 3\} \tag{16}$$

is optimal in an $N \times N$ duel. Then inequalities

$$u_{nj} = x_n^2 - y_j^2 - x_n^2 y_j^2 \geqslant 0 \ \ \forall\, y_j < x_n \ \text{ by } \ j = \overline{1,\ n-1} \tag{17}$$

and

$$u_{nj} = x_n^2 - y_j^2 + x_n^2 y_j^2 \geqslant 0 \ \ \forall\, y_j > x_n \ \text{ by } \ j = \overline{n+1,\ N} \tag{18}$$

must hold. From inequality (17) it follows that

$$\frac{x_n^2}{1 + x_n^2} \geqslant y_j^2 \ \ \forall\, y_j < x_n \ \text{ by } \ j = \overline{1,\ n-1}. \tag{19}$$

As

$$y_j \leqslant \frac{n-2}{N-1} < \frac{n-1}{N-1} = x_n$$

then inequality (19) is transformed into

$$\left(\frac{n-1}{N-1}\right)^2 \cdot \frac{1}{1 + \left(\frac{n-1}{N-1}\right)^2} \geqslant \left(\frac{n-2}{N-1}\right)^2,$$

$$(n-1)^2 \cdot \frac{1}{\frac{(N-1)^2+(n-1)^2}{(N-1)^2}} \geqslant (n-2)^2\,,$$

$$\frac{(n-1)^2\,(N-1)^2}{(N-1)^2+(n-1)^2} \geqslant (n-2)^2\,,$$

$$(n-1)^2\,(N-1)^2 \geqslant (n-2)^2\,(N-1)^2 + (n-2)^2\,(n-1)^2\,,$$

$$(N-1)^2\left[(n-1)^2-(n-2)^2\right] \geqslant (n-2)^2\,(n-1)^2\,,$$

$$(N-1)^2\,(2n-3) \geqslant (n-2)^2\,(n-1)^2\,,$$

$$(N-1)^2 \geqslant \frac{(n-2)^2\,(n-1)^2}{2n-3}. \tag{20}$$

From inequality (18) it follows that

$$1 \geqslant \frac{y_j^2-x_n^2}{x_n^2 y_j^2} = \frac{1}{x_n^2} - \frac{1}{y_j^2} \;\; \forall\, y_j > x_n. \tag{21}$$

As

$$1 \geqslant y_j > \frac{n-1}{N-1} = x_n \;\text{ for }\; n \in \left\{\overline{2,\; N-1}\right\}$$

then inequality (21) is transformed into

$$2 \geqslant \frac{1}{x_n^2} = \frac{(N-1)^2}{(n-1)^2},$$

whence

$$2\cdot(n-1)^2 \geqslant (N-1)^2\,. \tag{22}$$

Therefore, time moment (16) is optimal if inequalities (20) and (22) are true. This is equivalent to that membership

$$(N-1)^2 \in \left[\frac{(n-2)^2\,(n-1)^2}{2n-3};\; 2\cdot(n-1)^2\right] \tag{23}$$

is true for time moment (16). The closed interval in (23) is nonempty when its right endpoint is not less than its left endpoint:

$$2\cdot(n-1)^2 - \frac{(n-2)^2\,(n-1)^2}{2n-3} = (n-1)^2\left(2-\frac{(n-2)^2}{2n-3}\right) =$$

$$= (n-1)^2\left(\frac{4n-6-(n^2-4n+4)}{2n-3}\right) =$$

$$= (n-1)^2\left(\frac{-n^2+8n-10}{2n-3}\right) \geqslant 0. \tag{24}$$

Inequality (24) is true when

$$-n^2+8n-10 \geqslant 0,$$

i. e. when

$$n \in \left[4 - \sqrt{6};\ 4 + \sqrt{6}\right]. \tag{25}$$

So, time moment (16) can be optimal only for those integer n that satisfy (25), where

$$1.55 < 4 - \sqrt{6} < 1.56$$

and

$$6.44 < 4 + \sqrt{6} < 6.45,$$

i. e. there are only five possible cases of integer n:

$$n \in \{2,\ 3,\ 4,\ 5,\ 6\}. \tag{26}$$

Hence, the interval in (23) is nonempty if (26) is true.

For $n = 2$ the interval in (23) is $[0;\ 2]$ and membership (23) becomes equivalent to double inequality

$$0 \leqslant (N-1)^2 \leqslant 2,$$

whence

$$0 \leqslant N \leqslant \sqrt{2} + 1 < 3,$$

so the case of $n = 2$ is impossible. For $n = 3$ the interval in (23) is $\left[\frac{4}{3};\ 8\right]$ and membership (23) becomes equivalent to double inequality

$$\frac{4}{3} \leqslant (N-1)^2 \leqslant 8,$$

whence

$$\frac{2}{\sqrt{3}} + 1 \leqslant N \leqslant 2\sqrt{2} + 1 < 4,$$

so the case of $n = 3$ is impossible as well. For $n = 4$ the interval in (23) is $\left[\frac{36}{5};\ 18\right]$ and membership (23) becomes equivalent to double inequality

$$\frac{36}{5} \leqslant (N-1)^2 \leqslant 18,$$

whence

$$3 < \frac{6}{\sqrt{5}} + 1 \leqslant N \leqslant 3\sqrt{2} + 1 < 6$$

by

$$\{4,\ 5\} \subset \left[\frac{6}{\sqrt{5}} + 1;\ 3\sqrt{2} + 1\right],$$

so the case of $n = 4$ is possible for both $N = 4$ and $N = 5$. This means that in the 4×4 duel time moment $t_4 = 1$ is the single optimal one, whereas, without considering the duel end moment, in the 5×5 duel time moment $t_4 = \frac{3}{4}$ is the single optimal one.

For $n = 5$ the interval in (23) is $\left[\frac{144}{7};\ 32\right]$ and membership (23) becomes equivalent to double inequality

$$\frac{144}{7} \leqslant (N-1)^2 \leqslant 32,$$

whence

$$5 < \frac{12}{\sqrt{7}} + 1 \leqslant N \leqslant 4\sqrt{2} + 1 < 7,$$

by

$$5 \in \left[\frac{12}{\sqrt{7}} + 1;\ 4\sqrt{2} + 1\right],$$

so the case of $n = 5$ is possible for $N = 6$. This means that, without considering the duel end moment, in the 6×6 duel time moment $t_5 = \frac{4}{5}$ is the single optimal one.

For $n = 6$ the interval in (23) is $\left[\frac{400}{9};\ 50\right]$ and membership (23) becomes equivalent to double inequality

$$\frac{400}{9} \leqslant (N-1)^2 \leqslant 50,$$

whence

$$7 < \frac{23}{3} \leqslant N \leqslant 5\sqrt{2} + 1 < 9,$$

by

$$8 \in \left[\frac{23}{3};\ 5\sqrt{2} + 1\right],$$

so the case of $n = 6$ is possible for $N = 8$. This means that, without considering the duel end moment, in the 8×8 duel time moment $t_6 = \frac{5}{7}$ is the single optimal one.

If duel end moment $t_N = 1$ is optimal, then inequality

$$u_{Nj} = 1 - 2y_j^2 \geqslant 0 \ \ \forall\, y_j < 1 \ \text{ by } \ j = \overline{1,\ N-1} \tag{27}$$

must hold. Inequality (27) is rewritten as

$$y_j^2 = \frac{(j-1)^2}{(N-1)^2} \leqslant \frac{1}{2} \ \ \forall\, y_j < 1 \ \text{ by } \ j = \overline{1,\ N-1} \tag{28}$$

and inequality (28) is equivalent to inequality

$$y_{N-1}^2 = \frac{(N-2)^2}{(N-1)^2} \leqslant \frac{1}{2},$$

whence

$$2 \cdot (N-2)^2 \leqslant (N-1)^2,$$
$$2N^2 - 8N + 8 \leqslant N^2 - 2N + 1,$$
$$N^2 - 6N + 7 \leqslant 0. \tag{29}$$

Inequality (29) holds by

$$N \in \left[3 - \sqrt{2};\ 3 + \sqrt{2}\right],$$

i. e. when double inequality

$$1 < 3 - \sqrt{2} \leqslant N \leqslant 3 + \sqrt{2} < 5$$

holds, where

$$\{2,\ 3,\ 4\} \subset \left[3 - \sqrt{2};\ 3 + \sqrt{2}\right].$$

The cases of the duel end moment in the 3×3 and 4×4 duels have been already proved above. In bigger duels the end time moment cannot be optimal. This finalizes the proof.

6. Conclusion. An optimal time moment in quadratic-accuracy one-bullet silent duel (2) on the uniform lattice (6) by (3) — (5), (12) exists only if the duelist has three to eight time moments to shoot, except for seven time moments, and the optimal time moment is single. Compared to the linear-accuracy duel, which does not have an optimal time moment for six time moments and for no fewer than eight time moments, the quadratic-accuracy duel provides the duelist with a single one-step ("instant") solution for six and eight time moments instead of seven ones. Another difference is that the quadratic-accuracy duelist's optimal time moment gravitates more towards the duel end moment, unlike it is the duel span middle in the linear-accuracy duel. Thus, the quadratic-accuracy duelist's optimal time moment is the duel end moment in the 3×3 and 4×4 duels, is the penultimate time moment in the 5×5 and 6×6 duels, and is the time moment preceding the penultimate time moment in the 8×8 duel.

Conflict of Interest

The author declares that he has no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data Availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The author confirms that he did not use artificial intelligence technologies when creating the current work.

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Романюк В. В. Однокульова безшумна дуель із квадратичною точністю на рівномірній сітці: розв'язки в чистих стратегіях існують, коли дуелянт має не більше восьми моментів часу для пострілу.

Розглядається однокульова безшумна дуель, задана на рівномірній сітці часу, у якій кожен із двох дуелянтів здійснює постріл із квадратичною точністю. Дуель є симетричною матричною грою з нульовим значенням, причому обидва дуелянти мають однакову оптимальну поведінку як у чистих, так і в змішаних стратегіях. Завдання полягає у визначенні оптимальних моментів часу для здійснення пострілу залежно від кількості моментів часу в дуелі. Доведено, що оптимальний момент часу в дуелі існує лише тоді, коли дуелянт має від трьох до восьми моментів часу для пострілу, за винятком випадку семи моментів, причому оптимальний момент є єдиним. Для дуелі з квадратичною точністю оптимальним є кінцевий момент у дуелях 3×3 і 4×4, передостанній момент — у дуелях 5×5 і 6×6, а момент, що передує передостанньому, — у дуелі 8×8. Порівняно з дуеллю з лінійною точністю, у якій оптимальний момент відсутній при шести та при не менше ніж восьми моментах часу, дуель із квадратичною точністю забезпечує дуелянтові єдине однокрокове ("миттєве") розв'язання для шести та восьми моментів замість семи. Ще одна відмінність полягає в тому, що оптимальний момент дуелянта з квадратичною точністю тяжіє ближче до завершення дуелі, тоді як у дуелі з лінійною точністю він розташований у середині інтервалу дуелі.

Ключові слова: однокульова безшумна дуель, рівномірна сітка, квадратична точність, матрична гра, оптимальний момент часу.

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