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O. Yu. Masyutka[1], **M. P. Moklyachuk**[2]

[1] Taras Shevchenko National University of Kyiv,
Associate Professor of the Department of Mathematics and Theoretical Radiophysics,
Candidate of Physical and Mathematical Sciences
omasyutka@gmail.com
ORCID: https://orcid.org/0000-0002-7301-8813

[2] Taras Shevchenko National University of Kyiv,
Professor of the Department of Probability Theory, Statistics and Actuarial Mathematics ,
Doctor of Physical and Mathematical Sciences, Professor
moklyachuk@gmail.com
ORCID: https://orcid.org/0000-0002-6173-0280

ON ESTIMATION PROBLEM FOR MULTIDIMENSIONAL HOMOGENEOUS RANDOM FIELDS

The problem of the mean-square optimal estimation of the linear functionals which depend on the unknown values of a multidimensional homogeneous random field from observations of the field with noise is considered. Formulas for calculating the mean-square error and the spectral characteristic of the optimal linear estimate of the functionals are derived under the condition of spectral certainty, where the spectral densities of the fields are exactly known.

Keywords: multidimensional homogeneous random field, optimal estimate, mean square error, spectral density, spectral characteristic.

1. Introduction. Traditional methods of solution of the linear extrapolation, interpolation and filtering problems for stationary stochastic processes and homogeneous random fields are developed under the condition that spectral densities of processes and fields are known exactly (see, for example, selected works of A. N. Kolmogorov [7], survey by T. Kailath [5], books by Yu. A. Rozanov [13], E. J. Hannan [4], A. Malyarenko [8], V. S. Mandrekar and D. A. Redett [9], N. Wiener [15], M. I. Yadrenko [16], A. M. Yaglom [17, 18]).

The basic assumption of most of the methods of estimation of the unobserved values of stochastic processes and random fields is that the spectral densities of the considered stochastic processes and random fields are exactly known. However, in practice, these methods are not applicable since the complete information on the spectral densities is impossible in most cases. In order to solve the problem parametric or nonparametric estimates of the unknown spectral densities are found. Then, one of traditional estimation methods is applied, provided that the selected densities are the true ones. This procedure can result in significant increasing of the value of error as K. S. Vastola and H. V. Poor [14] have demonstrated with the help of some examples. To avoid this effect one can search the estimates which are optimal for all densities from a certain class of admissible spectral densities. These estimates are called minimax since they minimize the maximum value of the error. The paper by Ulf Grenander [3] should be marked as the first one where this approach to extrapolation problem for stationary processes was proposed. Several models of spectral uncertainty and minimax-robust methods of data processing can be found in the survey paper by S. A. Kassam and H. V. Poor [6]. In the paper by J. Franke [1] the minimax extrapolation problem for stationary sequences is investigated with the help of convex optimization methods. This approach makes it possible to find equations that determine the least favorable spectral densities for various classes of densities. In the book by M. Moklyachuk and A. Masyutka a minimax technique of the estimation for vector-valued stationary stochastic processes is proposed [10]. Estimation problems for random fields was considered in the paper by M. Moklyachuk and N. Shchestyuk [12]. In the book by M. P. Moklyachuk, O. Yu. Masyutka, I. I. Golichenko [11] the minimax approach is applied to investigate the estimation problems for functionals which depend on the unknown values of stationary random fields on a sphere.

In this paper we deal with the problem of the mean-square optimal linear estimation of the functionals

$$A_K\vec{\xi} = \iint\limits_K \vec{a}(s,t)^\top \vec{\xi}(s,t)dsdt,$$

which depend on the unknown values of a multidimensional homogeneous random field $\vec{\xi}(s,t) = \{\xi_k(s,t)\}_{k=1}^N$ from observations of the field $\vec{\xi}(s,t) + \vec{\eta}(s,t)$ at points $(s,t) \in \mathbb{R}^2 \setminus K$, where $\vec{\eta}(s,t) = \{\eta_k(s,t)\}_{k=1}^N$ is an uncorrelated with $\vec{\xi}(s,t)$ multidimensional homogeneous random field. Formulas for calculating the spectral characteristic and the mean square error of the optimal linear estimate of the functionals are derived under the condition that spectral densities of the fields are exactly known.

2. Mean-square optimal method of solution of the estimation problem for homogeneous random fields.

Let $\vec{\xi}(s,t) = \{\xi_k(s,t)\}_{k=1}^N$ and $\vec{\eta}(s,t) = \{\eta_k(s,t)\}_{k=1}^N$, $(s,t) \in \mathbb{R}^2$, be uncorrelated multidimensional homogeneous random fields with zero first moments $E\vec{\xi}(s,t) = \vec{0}$, $E\vec{\eta}(s,t) = \vec{0}$ and let the correlation functions

$$R_\xi(u,v) = E\vec{\xi}(s+u,t+v)(\vec{\xi}(s,t))^*,\ R_\eta(u,v) = E\vec{\eta}(s+u,t+v)(\vec{\eta}(s,t))^*$$

of the random fields $\vec{\xi}(s,t)$ and $\vec{\eta}(s,t)$ admit the spectral decomposition [9]

$$R_\xi(u,v) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} e^{i(u\lambda+v\mu)}F(\lambda,\mu)d\lambda d\mu,\ R_\eta(u,v) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} e^{i(u\lambda+v\mu)}G(\lambda,\mu)d\lambda d\mu,$$

where $F(\lambda,\mu) = \{f_{ij}(\lambda,\mu)\}_{i,j=1}^N$ and $G(\lambda,\mu) = \{g_{ij}(\lambda,\mu)\}_{i,j=1}^N$ are spectral density matrices of the fields $\vec{\xi}(s,t)$ and $\vec{\eta}(s,t)$, respectively.

Suppose that the spectral densities of fields satisfy the minimality condition

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\gamma(\lambda,\mu))^\top (F(\lambda,\mu)+G(\lambda,\mu))^{-1}\overline{\gamma(\lambda,\mu)}d\lambda d\mu < \infty, \tag{1}$$

$$\gamma(\lambda,\mu) = \iint\limits_K \vec{\alpha}(s,t)e^{i(s\lambda+t\mu)}dsdt.$$

Under this condition the error-free estimation is impossible [13].

The fields $\vec{\xi}(s,t)$ and $\vec{\eta}(s,t)$ admit the spectral decomposition [9]

$$\vec{\xi}(s,t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i(s\lambda+t\mu)}Z_{\xi}(d\lambda,d\mu),\ \vec{\eta}(s,t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i(s\lambda+t\mu)}Z_{\eta}(d\lambda,d\mu),\quad (2)$$

where $Z_{\xi}(d\lambda,d\mu)$ and $Z_{\eta}(d\lambda,d\mu)$ are vector-valued orthogonal stochastic measures of the fields $\vec{\xi}(s,t)$ and $\vec{\eta}(s,t)$.

Consider the problem of the mean-square optimal linear estimation of the functional

$$A_K\vec{\xi}=\iint\limits_{K}\vec{a}(s,t)^{\top}\vec{\xi}(s,t)dsdt,$$

which depend on the unknown values of the field $\vec{\xi}(s,t)$ from observations of the field $\vec{\xi}(s,t)+\vec{\eta}(s,t)$ at points $(s,t)\in\mathbb{R}^2\setminus K$.

Making use of spectral decomposition (2) of stationary field $\vec{\xi}(s,t)$ we can write the functional $A_K\vec{\xi}$ in the following form

$$A_K\vec{\xi}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(A_K(\lambda,\mu))^{\top}Z_{\xi}(d\lambda,d\mu),\ A_K(\lambda,\mu)=\iint\limits_{K}\vec{a}(s,t)e^{i(s\lambda+t\mu)}dsdt.$$

Denote by $\hat{A}_K\vec{\xi}$ the optimal linear estimate of the functional $A_K\vec{\xi}$ from observations of the field $\vec{\xi}(s,t)+\vec{\eta}(s,t)$ at points $(s,t)\in\mathbb{R}^2\setminus K$ and by $\Delta(F,G)=E\left|A_K\vec{\xi}-\hat{A}_K\vec{\xi}\right|^2$ the mean-square error of the estimate $\hat{A}_K\vec{\xi}$. Since spectral densities of stationary fields $\vec{\xi}(s,t)$ and $\vec{\eta}(s,t)$ are known, we can use the method of orthogonal projections in Hilbert spaces [7] to find the estimate $\hat{A}_K\vec{\xi}$.

Consider $\xi_k(s,t)$ and $\eta_k(s,t)$ as elements of the Hilbert space $H=L_2(\Omega,\mathcal{F},P)$ generated by random variables ξ with 0 mathematical expectations, $E\xi=0$, finite variations, $E|\xi|^2<\infty$, and the inner product $(\xi,\eta)=E\xi\overline{\eta}$. Denote by $H^{K-}(\xi+\eta)$ the closed linear subspace generated by elements $\{\xi_k(s,t)+\eta_k(s,t):(s,t)\in\mathbb{R}^2\setminus K\}$ in the Hilbert space $H=L_2(\Omega,\mathcal{F},P)$. Denote by $L_2(F+G)$ the Hilbert space of functions $\vec{a}(\lambda,\mu)$ such that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\vec{a}(\lambda,\mu))^{\top}(F(\lambda,\mu)+G(\lambda,\mu))\overline{\vec{a}(\lambda,\mu)}d\lambda d\mu<\infty,$$

and denote by $L_2^{K-}(F+G)$ the subspace of the space $L_2(F+G)$ generated by the functions

$$\{e^{i(s\lambda+t\mu)}\delta_k,\delta_k=\{\delta_{kl}\}_{l=1}^N,(s,t)\in\mathbb{R}^2\setminus K\}.$$

The mean-square optimal linear estimate $\hat{A}_K\vec{\xi}$ of the functional $A_K\vec{\xi}$ can be represented in the form

$$\hat{A}_K\vec{\xi}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(h(\lambda,\mu))^{\top}(Z_{\xi}(d\lambda,d\mu)+Z_{\eta}(d\lambda,d\mu)),$$

where $h(\lambda,\mu)\in L_2^{K-}(F+G)$ is the spectral characteristic of the estimate.

The mean-square error $\Delta(h;F,G)$ of the estimate $\hat{A}_T\vec{\xi}$ is given by the formula

$$\Delta(h;F,G)=E\left|A_K\vec{\xi}-\hat{A}_K\vec{\xi}\right|^2=$$

$$= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(A_K(\lambda,\mu)-h(\lambda,\mu))^\top F(\lambda,\mu)\overline{A_K(\lambda,\mu)-h(\lambda,\mu)}d\lambda d\mu+$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(h(\lambda,\mu))^\top G(\lambda,\mu)\overline{h(\lambda,\mu)}d\lambda d\mu.$$

According to the Hilbert space projection method proposed by A. N. Kolmogorov [7], the optimal estimation of the functional $A_K\vec{\xi}$ is a projection of the element $A_K\vec{\xi}$ of the space H on the space $H^{K-}(\xi+\eta)$. It can be found from the following conditions:

$$1)\hat{A}_K\vec{\xi}\in H^{K-}(\xi+\eta),$$
$$2)A_K\vec{\xi}-\hat{A}_K\vec{\xi}\perp H^{K-}(\xi+\eta).$$

It follows from the second condition that the spectral characteristic $h(\lambda,\mu)$ of the optimal linear estimate $\hat{A}_K\vec{\xi}$ for any $(s,t)\in\mathbb{R}^2\setminus K$ satisfies equations

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(A_K(\lambda,\mu)-h(\lambda,\mu)\right)^\top F(\lambda,\mu)e^{-i(s\lambda+t\mu)}d\lambda d\mu-$$
$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(h(\lambda,\mu))^\top G(\lambda,\mu)e^{-i(s\lambda+t\mu)}d\lambda d\mu=0.$$

The last relation is equivalent to equations

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}((A_K(\lambda,\mu))^\top F(\lambda,\mu)-(h(\lambda,\mu))^\top\times$$
$$\times(F(\lambda,\mu)+G(\lambda,\mu))e^{-i(s\lambda+t\mu)}d\lambda d\mu=0, \tag{3}$$

for any $(s,t)\in\mathbb{R}^2\setminus K$. Define the function

$$(C_K(\lambda,\mu))^\top=(A_K(\lambda,\mu))^\top F(\lambda,\mu)-(h(\lambda,\mu))^\top(F(\lambda,\mu)+G(\lambda,\mu))$$

and its Fourier transformation

$$\vec{c}(s,t)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}C_K(\lambda,\mu)e^{-i(s\lambda+t\mu)}d\lambda d\mu,\quad (s,t)\in\mathbb{R}^2.$$

It follows from the condition (3) that the function $\vec{c}(s,t)$ is nonzero only on the set K. Hence,

$$C_K(\lambda,\mu)=\iint\limits_K\vec{c}(s,t)e^{i(s\lambda+t\mu)}dsdt,$$

and the spectral characteristic of the estimate $\hat{A}_K\vec{\xi}$ is of the form

$$(h(\lambda,\mu))^\top=(A_K(\lambda,\mu))^\top F(\lambda,\mu)(F(\lambda,\mu)+G(\lambda,\mu))^{-1}-$$
$$-(C_K(\lambda,\mu))^\top(F(\lambda,\mu)+G(\lambda,\mu))^{-1}. \tag{4}$$

It follows from the first condition, $\hat{A}_K\vec{\xi}\in H^{K-}(\xi+\eta)$, which determine the optimal linear estimate of the functional $A_K\vec{\xi}$, that the following relation holds true for any $(s,t)\in K$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left((A_K(\lambda,\mu))^\top F(\lambda,\mu)(F(\lambda,\mu)+G(\lambda,\mu))^{-1}\right)e^{-i(s\lambda+t\mu)}d\lambda d\mu=$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left((C_K(\lambda,\mu))^\top (F(\lambda,\mu)+G(\lambda,\mu))^{-1}\right) e^{-i(s\lambda+t\mu)} d\lambda d\mu. \quad (5)$$

Let us define operators

$$(\mathbf{B}_K\mathbf{a})(s,t) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\iint\limits_K (\vec{a}(u,v))^\top (F(\lambda,\mu)+G(\lambda,\mu))^{-1}\times$$

$$\times e^{i(\lambda(u-s)+\mu(v-t))} dudvd\lambda d\mu,$$

$$(\mathbf{R}_K\mathbf{a})(s,t) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\iint\limits_K (\vec{a}(u,v))^\top F(\lambda,\mu)(F(\lambda,\mu)+G(\lambda,\mu))^{-1}\times$$

$$\times e^{i(\lambda(u-s)+\mu(v-t))} dudvd\lambda d\mu.$$

The relation (5) can be rewritten in the form

$$(\mathbf{R}_K\mathbf{a})(s,t) = (\mathbf{B}_K\mathbf{c})(s,t), \quad (s,t)\in K. \quad (6)$$

Suppose that the operator $\mathbf{B}_K$ is invertible. Then the function $\vec{c}(s,t)$ can be calculated by the formula

$$\vec{c}(s,t) = (\mathbf{B}_K^{-1}\mathbf{R}_K\mathbf{a})(s,t), \quad (s,t)\in K.$$

The mean-square error of the estimate $\hat{A}_K\vec{\xi}$ can be calculated by the formula

$$\Delta(F,G) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}((A_K(\lambda,\mu))^\top G(\lambda,\mu)+(C_K(\lambda,\mu))^\top)(F(\lambda,\mu)+G(\lambda,\mu))^{-1}\times$$

$$\times F(\lambda,\mu)(F(\lambda,\mu)+G(\lambda,\mu))^{-1}((A_K(\lambda,\mu))^\top G(\lambda,\mu)+(C_K(\lambda,\mu))^\top)^* d\lambda d\mu+$$

$$+\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}((A_K(\lambda,\mu))^\top F(\lambda,\mu)-(C_K(\lambda,\mu))^\top)(F(\lambda,\mu)+G(\lambda,\mu))^{-1}G(\lambda,\mu)\times$$

$$\times(F(\lambda,\mu)+G(\lambda,\mu))^{-1}((A_K(\lambda,\mu))^\top F(\lambda,\mu)-(C_K(\lambda,\mu))^\top)^* d\lambda d\mu. \quad (7)$$

Let us summarize results and present them in the form of a theorem.

Theorem 1. *Let $\vec{\xi}(s,t)$ and $\vec{\eta}(s,t)$ be uncorrelated multidimensional homogeneous random fields with spectral density matrices $F(\lambda,\mu)$ and $G(\lambda,\mu)$ which satisfy the minimality condition (1). The spectral characteristics $h(\lambda,\mu)$ and the mean-square error $\Delta(F,G)$ of the optimal linear estimate of the functional $A_K\vec{\xi}$ which depends on the unknown values of the field $\vec{\xi}(s,t)$ based on observations of the field $\vec{\xi}(s,t)+\vec{\eta}(s,t)$, $(s,t)\in\mathbb{R}^2\setminus K$ can be calculated by formulas (4), (7).*

Corollary 1. *Consider the estimation problem in the case where $K=\mathbb{R}\times[0,T]$. The spectral characteristics $h_T(F,G)$ of the optimal linear estimate of the functional*

$$A_T\vec{\xi} = \int_{-\infty}^{\infty}\int_0^T \vec{a}(s,t)^\top\vec{\xi}(s,t)dsdt,$$

which depends on the unknown values of the field $\vec{\xi}(s,t)$ based on observations of the field $\vec{\xi}(s,t)+\vec{\eta}(s,t)$, $(s,t)\in\mathbb{R}^2\setminus K$ can be calculated by formula (4) *where*

$$A_K(\lambda,\mu) = A_T(\lambda,\mu) = \int_{-\infty}^{\infty}\int_0^T \vec{a}(s,t)e^{i(s\lambda+t\mu)}dsdt = \int_0^T \vec{a}(\lambda,t)e^{it\mu}dt,$$

$$C_K(\lambda,\mu)=C_T(\lambda,\mu)=\int_{-\infty}^{\infty}\int_0^T\vec{c}(s,t)e^{i(s\lambda+t\mu)}dsdt=\int_0^T\vec{c}(\lambda,t)e^{it\mu}dt,$$

and $\vec{c}(\lambda,t)$ *is determined by relation*

$$\vec{c}(\lambda,t)=(\mathbf{B}_T^{-1}\mathbf{R}_T\mathbf{a})(\lambda,t),\quad (\lambda,t)\in\mathbb{R}\times[0,T], \tag{8}$$

where

$$(\mathbf{B}_T\mathbf{a})(\lambda,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_0^T(\vec{a}(\lambda,u))^\top(F(\lambda,\mu)+G(\lambda,\mu))^{-1}e^{i\mu(u-t)}d\mu du,$$

$$(\mathbf{R}_T\mathbf{a})(\lambda,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_0^T(\vec{a}(\lambda,u))^\top F(\lambda,\mu)(F(\lambda,\mu)+G(\lambda,\mu))^{-1}e^{i\mu(u-t)}d\mu du.$$

The mean-square error $\Delta_T(F,G)$ *of the optimal linear estimate of the functional* $A_T\vec{\xi}$ *can be calculated by formula*

$$\Delta_T(F,G)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\langle(\mathbf{R}_T\mathbf{a})(\lambda,t),(\mathbf{B}_T^{-1}\mathbf{R}_T\mathbf{a})(\lambda,t)\rangle+\langle(\mathbf{Q}_T\mathbf{a})(\lambda,t),\vec{a}(\lambda,t)\rangle d\lambda, \tag{9}$$

where

$$(\mathbf{Q}_T\mathbf{a})(\lambda,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_0^T(\vec{a}(\lambda,u))^\top F(\lambda,\mu)(F(\lambda,\mu)+G(\lambda,\mu))^{-1}\times$$

$$\times G(\lambda,\mu)e^{i\mu(u-t)}d\mu du,$$

and $<a,b>$ *is the inner product.*

Corollary 2. *Let* $\vec{\xi}(s,t)$ *be a multidimensional stationary stochastic field with the spectral density matrix* $F(\lambda,\mu)$*, which satisfies the minimality condition*

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\gamma(\lambda,\mu))^\top(F(\lambda,\mu))^{-1}\overline{\gamma(\lambda,\mu)}d\lambda d\mu<\infty, \tag{10}$$

$$\gamma(\lambda,\mu)=\iint\limits_K\vec{\alpha}(s,t)e^{i(s\lambda+t\mu)}dsdt.$$

The spectral characteristic $h_T(F)$ *and the mean-square error* $\Delta_T(F)$ *of the optimal linear estimate of the functional* $A_T\vec{\xi}$ *which depends on the unknown values of the field* $\vec{\xi}(s,t)$ *based on observations of the field* $\vec{\xi}(s,t)$ *at points* $(s,t)\in\mathbb{R}^2\setminus K$*,* $K=\mathbb{R}\times[0,T]$*, can be calculated by formulas*

$$(h_T(F))^\top=(A_T(\lambda,\mu))^\top-(C_T(\lambda,\mu))^\top(F(\lambda,\mu))^{-1}, \tag{11}$$

$$\Delta_T(F)=\frac{1}{2\pi}\int_{-\infty}^{\infty}<(\mathbf{B}_T^{-1}\mathbf{a})(\lambda,t),\vec{a}(\lambda,t)>d\lambda, \tag{12}$$

where

$$(\mathbf{B}_T\mathbf{a})(\lambda,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_0^T(\vec{a}(\lambda,u))^\top(F(\lambda,\mu))^{-1}e^{i\mu(u-t)}d\mu du.$$

3. Conclusions and prospects for further research. In this article we propose methods of the mean-square optimal linear estimation of the functionals which depend on the unknown values of the multidimensional homogeneous random field based on observed data of the field with noise. Under condition of spectral certainty where the spectral densities of the fields are exactly known we derive formulas for calculating the spectral characteristics and the mean-square errors of the estimates of the functionals. Analogous results are derived for the case of observations of the field without noise. The results of the minimax approach to estimation problem will be published in the next part of the article.

Conflict of Interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data Availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Contributions of authors

Mikhail Moklyachuk - methodology, derivation of the main results and preparation of the paper; Oleksandr Masyutka - derivation of the main results and preparation of the paper.

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Масютка О. Ю., Моклячук М. П. Задача оцінювання векторних однорідних випадкових полів.

Розглядається задача оптимального оцінювання лінійних функціоналів, що залежать від невідомих значень векторного однорідного випадкового поля за спостереженнями поля з шумом. Виведено формули для обчислення середньоквадратичної похибки та спектральної характеристики оптимальної лінійної оцінки функціоналів за умови спектральної визначеності, де спектральні щільності полів точно відомі.

Ключові слова: векторне однорідне випадкове поле, оптимальна оцінка, середньоквадратична похибка, спектральна щільність, спектральна характеристика.

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